Energy Efficient Heterogeneous Cellular Networks

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***Abstract*—With the exponential increase in mobile internet traffic driven by a new generation of wireless devices, future cellular networks face a great challenge to meet this overwhelming demand of network capacity. At the same time, the demand for higher data rates and the ever-increasing number of wireless users led to rapid increases in power consumption and operating cost of cellular networks. One potential solution to address these issues is to overlay small cell networks with macrocell networks as a means to provide higher network capacity and better coverage. However, the dense and random deployment of small cells and their uncoordinated operation raise important questions about the energy efficiency implications of such multi-tier networks. Another technique to improve energy efficiency in cellular networks is to introduce active/sleep (on/off) modes in macrocell base stations. In this paper, we investigate the design and the associated tradeoffs of energy efficient cellular networks through the deployment of sleeping strategies and small cells. Using a stochastic geometry based model, we derive the success probability and energy efficiency in homogeneous macrocell (single-tier) and heterogeneous K-tier wireless networks under different sleeping policies. In addition, we formulate the power consumption minimization and energy efficiency maximization problems, and determine the optimal operating regimes for macrocell base stations. Numerical results confirm the effectiveness of switching off base stations in homogeneous macrocell networks. Nevertheless, the gains in terms of energy efficiency depend on the type of sleeping strategy used. In addition, the deployment of small cells generally leads to higher energy efficiency but this gain saturates as the density of small cells increases. In a nutshell, our proposed framework provides an essential understanding on the deployment of future green heterogeneous networks.**

***Index Terms*—Energy efficiency, green communications, heterogeneous wireless networks, power consumption, sleeping strategy, small cells, open access, stochastic geometry.**

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I. Introduction

EXISTING cellular architectures are designed to cater to large coverage areas, which often fail to achieve the expected throughput to ensure seamless mobile broadband in the uplink as users move far away from the base station. This is mainly due to the increase in inter-cell interference, as well as constraints on the transmit power of the mobile devices. Another limitation of conventional macrocell approach is the poor indoor penetration and the presence of dead spots, which result in drastically reduced indoor coverage. In order to overcome these issues and provide a significant network performance leap, heterogeneous networks have been introduced in the LTE-Advanced standardization [1]–[3]. A heterogeneous network uses a mixture of macrocells and small cells such as microcells, picocells, and femtocells. These small cells can potentially improve spatial reuse and coverage by allowing future cellular systems to achieve higher data rates, while retaining seamless connectivity and mobility in cellular networks.

Besides the issue of meeting overwhelming traffic demands, network operators around the world now realize the importance of managing their cellular networks in an energy efficient manner and reducing the amount of CO_2 emission levels simultaneously [4]–[7]. Current studies show that the amount of CO_2 emission levels due to information and communication technologies is already 2%. With the exponential increase in data traffic and mobile devices, this figure is projected to increase significantly. Improving energy efficiency also helps network operators reduce the operational cost as energy constitutes a significant part of their expenditure. As a result, the terminology of "green cellular network" has become very popular recently, showing the current sentiment of the telecom industries to place more emphasis on energy efficiency as one of the key performance indicators for cellular network design [7].

Although the deployment of small cell networks is seen to be a promising way of catering to the ever increasing traffic demands, the dense and random deployment of small cells and their uncoordinated operation raise important questions about the implication of energy efficiency in such multi-tier networks [8]–[11]. Besides introducing small cells into existing macrocell networks, another effective technique is to introduce sleep mode in macrocell base stations (MBSs) [12]–[15]. The main motivation is that current cellular networks usually assume that the traffic demand is always high and so the MBSs are always powered on at all times. However, studies have shown that there are high fluctuations in traffic demand over space and time in cellular networks [6]. For example, the traffic demands in urban and rural areas or traffic demands in day and night time are entirely different. From this perspective, there is potential in energy savings by adapting the sleeping mode of MBSs to the demanded traffic. Nevertheless, when we switch some MBSs off, certain users may need to connect to MBSs located further away while experiencing a lower amount of intercell interference. For the case of dense deployment of MBSs, we know that these two effects cancel out equally and the coverage probability is independent of the sleeping mode [16]. However, for sparse deployment of MBSs, it is expected that we need to maintain the coverage of the cellular networks when we implement sleeping mode in MBSs either through power control or open access small cells. Since both techniques consume power, it is unclear which technique is more energy efficient and how the energy efficiency depends on the intensity of small cells and access policy.

On the other hand, one of the major challenges in small cell deployment is the incursion of inter-tier interference due to aggressive frequency reuse, which can deteriorate the effectiveness of small cell architecture [1]–[3]. As a result, there has been a significant amount of research on managing inter-tier and intra-tier interference in a two-tier small cell network, which consists of a macrocell network overlaid with small cells [17], [18]. In [17], the authors proposed a spectrum partitioning approach to avoid the inter-tier interference between the macrocell and small cell tiers by using orthogonal spectrum allocation. However, under a sparse small cell deployment setting, this approach is clearly inefficient and much higher area spectrum efficiency can be attained if spectrum sharing is allowed [18]. On the other hand, for spectrum sharing in two-tier small cell networks, it becomes imperative to properly manage the inter-tier interference using techniques such as access control [18], [19], power control [20], [21], multiple antennas [22], or cognitive radio [23]–[25]. Besides interference management techniques, interference modeling in two-tier networks using stochastic geometry has also gathered considerable attention due to its accuracy and tractability [26]–[28]. The spatial distribution of MBSs in the network is usually modeled by lattices or hexagonal cells since their deployment is considered well-planned, centralized, and hence regular. Nevertheless, it has been recently shown that modeling MBSs by a homogeneous Poisson point process (PPP) and associating macrocell users to their closest MBSs is a tractable yet accurate macrocell network model [16]. On the other hand, femtocell access points (FAPs) are extensively modeled as PPP as well, mainly due to uncoordinated and random deployment and operation.

In this work, we apply the tools from stochastic geometry to analyze the energy efficiency of cellular networks through the deployment of sleeping strategy as well as small cells. By assuming that the network operators have some information of the traffic usage patterns, they can employ a coordinated sleeping mode, where certain MBSs will be shut off while others increase their coverage areas to avoid coverage hole. In particular, we model the sleeping mode at each MBS as a Bernoulli random variable, where q denotes the probability that a MBS remains in operation and the underlying spatial distribution of MBSs is modeled as a PPP. In practice the network operators will have a predetermined policy of sending MBSs to sleep that ensures reasonable coverage over the entire network, i.e., such as spacing out sleeping MBSs regularly. We nevertheless adopt a marked PPP to model the dynamics of the sleeping mode (which is a random process) for its tractability in order to come up with reasonable design guidelines of green cellular network design. To maintain similar network coverage after some MBSs have been switched off, we need to perform some form of power control. Given no knowledge of the channel state information, we will employ fixed power control. One question we will explore is the effect that q has on the energy efficiency when we shut some MBSs off. While we will reduce the interference from some MBSs, this will cause certain macrocell users to connect to MBSs which are even further away. Besides homogeneous macrocell networks with sleeping strategy, we will also investigate the energy efficiency in heterogeneous K-tier networks with open access small cells. In addition, we formulate optimization problems in the form of power consumption minimization and energy efficiency maximization and determine the optimal operating frequency of the macrocell base station. Numerical results confirm that the effectiveness of sleeping strategy in homogeneous macrocell networks but the gain in energy efficiency depends on the type of sleeping strategy used. In addition, the deployment of small cells generally lead to higher energy efficiency but this gain saturates as the density of small cells increases.

The remainder of this paper is organized as follows: In Section II, we present our system model. In Section III, we analyze a homogeneous macrocell networks with random and strategic sleeping policies. In Section IV, we extend these results to heterogeneous K-tier networks. In Section V, we validate our analysis through simulation results, and concluding remarks are given in Section VI.

II. System Model

A. Network Model

We consider a wireless cellular network consisting of MBSs located according to a homogeneous PPP Θ_{M} of intensity λ_{M} in the Euclidean plane. Users are distributed according to a different independent stationary point process of intensity μ. Each macrocell user is associated with its geographically closest MBS and the analysis is performed for a randomly selected typical user. Since Θ_{M} is a stationary process, the distribution of distance R_{M} between a macrocell user and its designated MBS remains the same regardless of the exact locations, and its probability density function (pdf) is given by $f_{R_M}(r) = 2\pi\lambda_{\mathrm{M}} r \exp(-\lambda_{\mathrm{M}}\pi r^2)$. We assume universal frequency reuse among base stations and that each MBS serves only one user. If there are multiple users in a Poisson-Voronoi cell, some form of orthogonal resource sharing (e.g. frequency or time division multiple access) is performed.

B. Signal-to-Interference-plus-Noise Ratio

For notational convenience, we denote a base station by its location while the user is at the origin 0. For downlink transmission of a MBS x to the typical user 0, the *signal-to-*

interference-plus-noise ratio (SINR) experienced by a macrocell user is given by

$$\mathsf{SINR}_{\mathrm{M}}(x \to u) = \frac{P_{t,i} h_x g(x)}{\sum_{y \in \Theta(x)} P_{t,y} h_y g(y) + \sigma^2} \tag{1}$$

where $\Theta(x)$ denotes the set of nodes interfering with x, $P_{t,i}$ denotes the transmit power at tier i, and h_x, h_y are the channel power gain due to small-scale fading from x, y respectively. In the following, we assume that $h_x \sim \exp(1)$ and $h_y \sim \exp(1)$ (Rayleigh fading). The background noise is assumed to be additive white Gaussian with variance σ^2 and the path loss function is denoted by $g(x) = \|x\|^{-\alpha}$, with α being the path loss exponent.

C. Performance Metrics

Using (1) we can define the *success probability* from x to u as $\mathbb{P}(\mathsf{SINR}_{\mathrm{M}}(x \to u) > \gamma)$, where γ is a prescribed quality-of-service (QoS) threshold. By averaging the success probability over the distance to the nearest node, we obtain the *coverage probability* of a typical macrocell user given by $\mathbb{P}_{\mathrm{M}}(\gamma)$. The throughput attained at a given BS-user link is given by $\mathbb{P}(\mathsf{SINR} > \gamma) \log_2(1+\gamma)$ and the area spectral efficiency (network throughput) is taken over all the links in the network, where for a homogeneous network scenario is defined as $\mathcal{T}_{\mathrm{M}} = \lambda_{\mathrm{M}} \mathbb{P}_{\mathrm{M}}(\gamma) \log_2(1+\gamma)$. Lastly, we define the energy efficiency E_{eff} as follows:

$$E_{\text{eff}} = \frac{\text{Area Spectral Efficiency}}{\text{Average Network Power Consumption}} = \frac{\mathcal{T}}{\lambda_{\mathrm{M}} P_{\text{tot}}}, \tag{2}$$

where P_{tot} denotes the MBS power consumption.

D. Power Consumption Model

1) Homogeneous (Single-tier) Network model: The power consumption at each MBS is given by $P_{\text{tot}} = P_{\mathrm{M}0} + \beta \Delta_{\mathrm{M}} P_{\mathrm{M}}$ where $P_{\mathrm{M}0}$ is the static power expenditure of the MBS, βP_{M} is the RF output power of the MBS, and Δ_{M} is the slope of the load-dependent power consumption in MBS [6]. A fixed power control policy is adopted here in order to avoid creating *coverage holes* or areas where the target SINR is below an acceptable level due to switching off MBS. To ensure a similar level of coverage as before sleeping, we assume that all awake MBS transmit with power βP_{M}, where β is a ratio that represents power control. It is assumed that β is the same for all MBSs.

2) K-tier Heterogeneous Network model: In the second part of the paper, we also consider a general K-tier heterogeneous network model, where the base stations in each tier are modeled as independent homogeneous PPP Θ_i with intensity λ_i. We will always use Θ_1 for the macro tier Θ_{M}. In addition, we consider again that all base stations in the K tiers share the same bandwidth. Without employing any sleeping mode at each base station in the i-th tier, the average power consumption of the i-th tier heterogeneous networks is given by

$$P_{\text{Het},i} = \lambda_i (P_{i0} + \Delta_i P_i), \tag{3}$$

where P_{i0} is the static power expenditure of the base station in the i-th tier, P_i is the RF output power of the i-th tier base station, and Δ_i is the slope of the load-dependent power consumption the base station in the i-th tier.

E. Base Station Sleep Mode Strategies

In this section, we present the two main policies that we propose and analyze in order to optimize the power consumption at each MBS. We investigate policies of dynamically switching off MBS, where the power consumed by a switched off MBS in sleep mode is P_{sleep}.[1] To maintain similar network coverage after some MBSs having been switched off, we employ power control by selecting $P_{t,\mathrm{M}} = \beta P_{t_0,\mathrm{M}}$, where β denotes the uniform increase in transmission power for MBS. The attractiveness of fixed power control is that it compensates for the sleeping activity without the need for obtaining instantaneous channel state information for the macrocell users.

1) Random Sleeping: In *random sleeping*, we model the sleeping strategy as a Bernoulli trial such that each station continues to operate with probability q and *sleeps* (is turned off) with probability $1-q$, independently of all the other base stations. Therefore, after applying random sleeping at the macro tier, the average total power consumption of the macrocell network is given by

$$P_{\text{RS}} = \lambda_{\mathrm{M}} q (P_{\mathrm{M}0} + \Delta_{\mathrm{M}} \beta P_{\mathrm{M}}) + \lambda_{\mathrm{M}} (1-q) P_{\text{sleep}}. \tag{4}$$

2) Strategic Sleeping: Instead of randomly switching MBSs off, we can also switch off MBSs when their activity levels are low, e.g. when load or traffic demands are low. Specifically, we model this strategic sleeping as a function $s : [0,1] \mapsto [0,1]$ which says that if the activity level of the coverage area associated with the MBS has activity level x, then it operates with probability $s(x)$ and sleeps with probability $1 - s(x)$, independently. This sleep mode strategy can be seen as a load-aware policy and it can incorporate traffic profile in the optimization problem. As a result, the average power consumption of the macrocell network after employing strategic sleeping is given by

$$P_{\text{SS}} = \lambda_{\mathrm{M}} \mathbb{E}\{s\} (P_{\mathrm{M}0} + \beta \Delta_{\mathrm{M}} P_{\mathrm{M}}) + \lambda_{\mathrm{M}} (1 - \mathbb{E}\{s\}) P_{\text{sleep}}, \tag{5}$$

where $\mathbb{E}\{s\} = \int_0^1 s(x) f_A(x) \mathrm{d}x$ and $f_A(x)$ is the pdf of A and A denotes the random activity within a cell and takes values in $[0,1]$. The rationale behind the proposed strategic sleeping is the following: while random sleeping models a network that is adaptive to the fluctuating activity levels during the day, strategic sleeping goes one step further and models a network that is adaptive to the fluctuating activity levels within the location. Furthermore, the strategic sleeping model may be used as a method of measuring the impact of cooperation among MBSs. Let us illustrate this with an example. Suppose that we have a pair of cooperating MBSs. If the activity level in the combined coverage area is expected to be below half of the full capacity, then the pair may choose to keep only one of them awake. Then, the awake MBS may serve both coverage areas or the coverage areas can be reassigned among all remaining awake MBSs. The above cooperation model can be modeled by strategic sleeping by having, say, both MBS to stay awake with probability $s = 0.5$. While an explicit

[1]Note that we consider that $P_{\text{sleep}} < P_{\mathrm{M}0}$ which is a valid assumption for future base stations with sleeping mode capabilities.

association between neighboring MBSs is technically absent, this model may nevertheless be seen as a way to measure the energy savings by introducing cooperation within the network.

III. Homogeneous Macrocell Network

In this section, we study the effect of switching off MBSs based on the aforementioned sleeping policies, i.e. randomly and dynamically. The performance measure is the coverage probability and the effect of noise is taken into account, i.e. $\sigma^2 > 0$. In recent work analyzing coverage in macrocellular networks, it is shown that the coverage probability is independent of the intensity of the base stations in the interference-limited regime ($\sigma^2 \to 0$) [16]. This also holds true in heterogeneous K-tier networks [18], [29]. The main reason behind this is the fact that in dense networks, the improvement in received signal power by adding more MBSs and bringing the transmitters closer to the receivers is equally canceled out by the increased interference from more MBSs (interferers). Nevertheless, when MBS sleeping policies are applied, the effect of noise is noticeable and cannot be ignored as the number of interferers may be significantly decreased. Therefore, in this work we also consider the case where $\sigma^2 > 0$.

A. Random Sleeping

As explained in Section II, the random sleeping strategy is simply equivalent to modeling the active MBSs as a marked PPP with intensity $q\lambda_{\rm M}$ and increasing the transmission power of the active MBSs to βP_M.

Theorem 1. *In homogeneous macrocell networks with random sleeping, the coverage probability of a randomly located macrocell user is given by*

$$\mathbb{P}_{\rm RS}(\beta,\gamma) = 2\pi q\lambda_{\rm M}\int_{r=0}^{\infty} r\exp(-\pi r^2 q\lambda_{\rm M}(1+\rho(\gamma,\alpha))) \times \exp(-r^\alpha\gamma\sigma^2/\beta P_{t,{\rm M}}){\rm d}r \quad (6)$$

where $\rho(\gamma,\alpha) = \gamma^{2/\alpha}\int_{\gamma^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du$.
Furthermore, for $\sigma^2 = 0$, $\mathbb{P}_{\rm RS}(\beta,\gamma)$ *can be simplified as*

$$\mathbb{P}_{\rm RS}(\beta,\gamma) = \frac{1}{1+\rho(\gamma,\alpha)}. \quad (7)$$

Proof: See Appendix B. □

From Theorem 1, we can see that the coverage probability is completely independent of the sleeping policy, the density of MBSs $\lambda_{\rm M}$, as well as the power control β when $\sigma^2 = 0$. The only parameter that affects the coverage probability is the target SINR threshold γ. In the case of $\sigma^2 > 0$, numerical integration is required to calculate the coverage probability.

B. Strategic Sleeping

Here we analyze the strategic MBS switching off that is based on the activity of macrocell users in each cell. We assign i.i.d. random variables $A_i \sim A$ to each MBS $i \in \Theta_{\rm M}$, such that A takes values in $[0,1]$. A_i represents user activity within the Poisson-Voronoi cell that the MBS covers. That is to say, for any user located in a Poisson-Voronoi cell of a MBS with activity level a, the user is *active* with probability a, i.e. it is actually connected to the MBS with probability a. Therefore, we can model the sleeping strategy as a function $s:[0,1]\mapsto[0,1]$, which implies that if the activity level of the MBS has activity level x, then it operates with probability $s(x)$ and sleeps with probability $1-s(x)$. In addition, we impose that $s(x)$ is increasing. Using this model, the active MBSs are distributed accordingly to a homogeneous PPP with intensity $\lambda_{\rm M}\mathbb{E}\{s\} = \lambda_{\rm M}\int_0^1 s(x)f_A(x){\rm d}x$. Therefore, the coverage probability that captures the activity of the macrocell user is provided in the next theorem.

Theorem 2. *The coverage probability of the active macrocell user*[2] *is given by*

$$\begin{aligned}&\mathbb{P}_{\rm SS}(\beta,\gamma)\\ &= \frac{1}{\mathbb{E}\{a\}}\Bigg\{\int_0^1 xs(x)f_A(x){\rm d}x\\ &\int_{r=0}^{\infty}\exp(-\pi r^2\lambda_{\rm M}\mathbb{E}\{s\}\rho(\gamma,\alpha))\exp(-r^\alpha\sigma^2\gamma/\beta P_{t,{\rm M}})g_1 r{\rm d}r\\ &+\int_0^1 x(1-s(x))f_A(x){\rm d}x\sum_{i=2}^{\infty}\int_{r=0}^{\infty}\exp(-r^\alpha\sigma^2\gamma/\beta P_{t,{\rm M}})\\ &\exp(-\pi r^2\lambda_{\rm M}\mathbb{E}\{s\}\rho(\gamma,\alpha))g_i r{\rm d}r\Bigg\} \quad (8)\end{aligned}$$

where $g_i(r)$ *is the pdf of the* i*-th nearest point from a PPP, such that* $g_i(r) = \frac{2\pi^i r^{2i-1}\lambda_{\rm M}^i}{(i-1)!}\exp(-\pi r^2\lambda_{\rm M})$. *For* $\sigma^2 = 0$, $\mathbb{P}_{\rm SS}(\beta,\gamma)$ *can be simplified as*

$$\mathbb{P}_{\rm SS}(\beta,\gamma) = \frac{1+\rho(\gamma,\alpha)\mathbb{E}\{as(a)\}/\mathbb{E}\{a\}}{(1+\mathbb{E}\{s\}\rho(\gamma,\alpha))(1+\rho(\gamma,\alpha))}. \quad (9)$$

Proof: See Appendix C. □

For the case of $\sigma^2 = 0$, we can see that the coverage probability is independent of the intensity of MBSs and the transmit power. Unlike the case of random sleeping, the strategic sleeping has an effect on the coverage probability even in the interference-limited regime ($\sigma^2 = 0$). Using (9), which corresponds to the interference-limited regime, we can show an interesting property of the strategic sleeping: the coverage probability of the active macrocell user is at least as good as in the case where no sleeping mode is employed.

Lemma 1. *When* $\sigma^2 = 0$*, the sleeping strategy* s *improves the coverage probability of the active macrocell user if it satisfies the following inequality*

$$\mathbb{E}\{as(a)\} > \mathbb{E}\{s\}\mathbb{E}\{a\}. \quad (10)$$

Proof: The proof is omitted as it follows after standard algebraic manipulations. □

The consequence of Lemma 1 is that, for fixed $\mathbb{E}\{s\}$, if we want to maximize $\mathbb{E}\{as(a)\}$, we need to match large values of s with high activity. Thus, by assuming that $s(x)$ is increasing, this guarantees that strategic sleeping cannot result in worse performance than the case of no switching off as stated in the following lemma.

[2]The key difference in this metric is that we obtain the average probability of a successful transmission *conditioned* on that the user is *active*. Our motivation for doing so is that we think that the number of mobile users in a certain location may be very small at certain times of the day and it may not be fair to give these locations the same consideration as other locations where it is known that there is a large number of mobile users.

Lemma 2. *When $\sigma^2 = 0$, if $s(x)$ is increasing in x, the coverage probability of the active macrocell user in the strategic sleeping case is at least as good as the non sleeping case.*

Proof: The result is a consequence of a more general result that states that, given two increasing measurable functions on a random variable, the covariance is non-negative. The proof can be found in [30]. □

Therefore, from Lemma 1, we conclude that a strategic, load-aware sleeping policy suggests the intuitive policy that a high fraction of MBSs is switched off when the activity is low. This means that users in areas with low activity are heavily penalized. However, Lemma 2 assures us that the benefits for the majority of the users outweighs the decreased performance for the minority.

Remark 1. *The above results can be easily extended to the case where base stations and users have multiple antennas. The main technical challenge is the fact that the small-scale fading variables will be in general distributed according to gamma distribution (chi-squared) with different shape and scale parameters, depending on the multi-antenna scheme used. Briefly, using properties of the Laplace transform, we can show that this will make appear higher order derivatives of the Laplace transform of the interference in the above expressions for the success probability and throughput. Detailed investigation on the effect of multiple antennas on the energy efficiency is left for future work.*

C. Constrained Optimization Framework

In the following, we use the results from the previous section to solve several energy efficiency related optimization problems under different sleeping policies.

1) Power Consumption Minimization with Random Sleeping: In the first problem, we minimize the power consumption subject to a coverage probability constraint, which can be interpreted as a QoS constraint. In the case of random sleeping, the problem is formulated as follows

$$\mathcal{P}_{\mathrm{RS}} : \begin{cases} \min\limits_{q} & \lambda_{\mathrm{M}} q (P_{\mathrm{M}0} + \Delta_M \beta P_{\mathrm{M}}) \\ & +\lambda_{\mathrm{M}}(1-q)P_{\mathrm{sleep}} \\ \text{s.t.} & \mathbb{P}_{\mathrm{RS}}(\beta,\gamma) \geq \epsilon \end{cases} \tag{11}$$

where q is the fraction of MBSs that are still operating. In order to solve the above problem, we first show that the coverage probability is an increasing function of a certain variable x. Then, we find the value $x^\star$ that satisfies the constraint tightly, and finally, we solve the minimization problem subject to the condition $x^\star$. Therefore, rewriting $q\lambda_{\mathrm{M}} = S$ in Theorem 1, we have

Lemma 3. *For $\sigma^2 > 0$ and $\alpha > 2$, the coverage probability $\mathbb{P}_{\mathrm{RS}}$ increases with increasing S.*

Proof: By rewriting the success probability, we have $S\int_{r=0}^{\infty} 2\pi r \exp(-r^2 S c_1)\exp(-r^\alpha c_2)\mathrm{d}r = S\int_{x=0}^{\infty} 2\pi \exp(-xSc_1)\exp(-x^{\alpha/2}c_2)\mathrm{d}x$.
Let $T > S$ and by substituting $y = xT/S$, we obtain $T\int_{x=0}^{\infty} 2\pi \exp(-xTc_1)\exp(-x^{\alpha/2}c_2)\mathrm{d}x = S\int_{y=0}^{\infty} 2\pi\exp(-ySc_1)\exp(-y^{\alpha/2}(S/T)^{\alpha/2}c_2)\mathrm{d}x > S\int_{y=0}^{\infty} 2\pi \exp(-ySc_1)\exp(-y^{\alpha/2}c_2)\mathrm{d}x$. The last step makes use of the fact $\alpha > 2$ and $c_2 = \sigma^2\gamma > 0$. Hence, we have $\mathbb{P}_{\mathrm{RS}}$ increases with S. □

From Lemma 3, we may conclude that the minimum power consumption occurs when $q\lambda_{\mathrm{M}}$ satisfies the constraint tightly. Hence, $q^\star_{\mathrm{PC,RS}}$ is given by

$$\epsilon = 2\pi q^\star_{\mathrm{PC,RS}}\lambda_{\mathrm{M}} \int_{r=0}^{\infty} r\exp(-\pi r^2 q^\star_{\mathrm{PC,RS}}\lambda_{\mathrm{M}}(1+\rho(\gamma,\alpha))) \exp(-r^\alpha\gamma\sigma^2/\beta P_{t,\mathrm{M}})\mathrm{d}r. \tag{12}$$

2) Power Consumption Minimization with Strategic Sleeping: The minimization problem in the case of strategic sleeping is formulated similarly as

$$\mathcal{P}_{\mathrm{SS}} : \begin{cases} \min\limits_{s} & \lambda_{\mathrm{M}}(\mathbb{E}\{s\})(P_{\mathrm{M}0} + \Delta_M\beta P_{\mathrm{M}}) \\ & +\lambda_{\mathrm{M}}(1-\mathbb{E}\{s\})P_{\mathrm{sleep}} \\ \text{s.t.} & \mathbb{P}_{\mathrm{SS}}(\beta,\gamma) \geq \epsilon. \end{cases} \tag{13}$$

Solving the above optimization problem is more challenging in the case of strategic sleeping since before stating that the constraint is satisfied by equality, we first need to compute the optimal strategy as shown in the following lemma.

Lemma 4. *For a fixed $\mathbb{E}\{s\}$, the strategy that optimizes the success probability per active user is to have $s(a) = 1_{\{a\geq a_0\}}(a)$ for some a_0. That is to say, the strategy takes a form of a threshold function where the MBS is switched on if the activity exceeds a_0.*

Proof: See Appendix D. □

Therefore, the optimal solution $s^\star(a)$ can be characterized by a single variable a_0, which we denote as $a^\star$. The optimization problem is solved using equality for the QoS constraint, in which case, the solution is characterized based on $a^\star$.

Theorem 3. *The optimal $s^\star(a)$, denoted as $a^\star$, satisfies*

$$\epsilon = \frac{1}{\mathbb{E}\{a\}}\Big\{\mathbb{P}(\mathsf{SINR} > \gamma | N_{\mathrm{ord}} = 1)\int_{a^\star}^{1} x f_A(x)\mathrm{d}x + \mathbb{P}(\mathsf{SINR} > \gamma | N_{\mathrm{ord}} > 1)\int_{0}^{a^\star} x f_A(x)\mathrm{d}x\Big\} \tag{14}$$

where

$$\mathbb{P}(\mathsf{SINR} > \gamma | N_{\mathrm{ord}} = 1) = \int_{r=0}^{\infty} e^{(-\pi r^2\lambda_{\mathrm{M}}\mathbb{E}\{s\}\rho(\gamma,\alpha))} e^{(-r^\alpha\sigma^2\gamma/\beta P_{t,\mathrm{M}})} g_1 r\mathrm{d}r,$$

and

$$\mathbb{P}(\mathsf{SINR} > \gamma | N_{\mathrm{ord}} > 1) = \sum_{i=2}^{\infty}\int_{r=0}^{\infty} e^{(-\pi r^2\lambda_{\mathrm{M}}\mathbb{E}\{s\}\rho(\gamma,\alpha))} e^{(-r^\alpha\sigma^2\gamma/\beta P_{t,\mathrm{M}})} g_i r\mathrm{d}r.$$

$N_{\mathrm{ord}} = i$ denotes the event the user is connected to the $i-$th nearest MBS.

Despite the simple form of the optimal strategy, which is to switch on MBSs when the activity level exceeds a threshold, it may be realistic to assume a probabilistic decision making function taking probabilities that are not in $\{0, 1\}$. This is because operators may choose to shut down MBSs in a coordinated fashion according to the activity in a certain location. While this does not model coordination between neighboring cells, we can use intermediate probabilities to model the effect of coordination with a neighboring MBS which the current MBS hands traffic over to.

IV. Heterogeneous K-Tier Networks

In the following, we consider that all base stations in the heterogeneous networks operate in *open access*, i.e. any user is allowed to connect to access points (called below as BSs) from any tier [29]. We consider three different user association schemes, namely *location based scheme*, *average signal based scheme*, and *instantaneous SINR based scheme*. Specifically, we have

- *Location based scheme:* Assume that a user knows the locations of nearby access points from all tiers and its own location. The user computes the relative distance to the nearest access points from each tier, which we denote as $\{r_i\}$. Define a biasing system, which are real numbers $\{\kappa_i\}$. The user then connects to the BS corresponding to $\min(\{\kappa_i r_i\})$.
- *Average signal based scheme:* Assume that users are able to associate with the BS based on the perceived *average* SINR. Denote $\{Q_i\}$ as the highest perceived average signal from each tier. Similarly define a biasing system $\{\tau_i\}$. The user connects to the BS corresponding to $\max(\{\tau_i Q_i\})$.
- *Instantaneous SINR based scheme:* This model is based on [29]. The user connects to tier i if the instantaneous SINR exceeds γ_i. We assume that $\gamma_i > 1$ so at most one tier will provide a signal exceeding the threshold, in which case we say that the user is *connected*.

We first provide the coverage probability for the location based scheme:

Theorem 4. *The coverage probability for the general mobile user operating under the location based scheme is given by*

$$\mathbb{P}_{\text{LOC}} = \sum_{i=1}^{K} \int_{r=0}^{\infty} 2\lambda_i \pi r \exp(-r^2 c_i) \exp(-r^\alpha a_i) \mathrm{d}r \quad (15)$$

where $a_i = \gamma\sigma^2/P_{t,i}$ *and* $c_i = \pi\lambda_i(1 + \rho(\gamma, \alpha)) + \frac{\pi}{\kappa_i^2} \sum_{j \neq i} \lambda_j \big(1 + \rho\big(\gamma \frac{P_{t,j}}{P_{t,i}} \frac{\kappa_i^\alpha}{\kappa_j^\alpha}, \alpha\big)\big)$. *When* $\sigma^2 = 0$ *we have*

$$\mathbb{P}_{\text{LOC},\sigma^2=0} = \sum_{i=1}^{K} \frac{\lambda_i \kappa_i^2}{\sum_j \lambda_j \kappa_j^2 (1 + \rho\big(\gamma \frac{P_{t,j}}{P_{t,i}} \frac{\kappa_i^\alpha}{\kappa_j^\alpha}, \alpha\big))}. \quad (16)$$

Proof: See Appendix E. □

Instead of deriving the coverage probability for the average signal based scheme, we show that the *location based* and the *average signal based* schemes are equal with an appropriate choice of biasing factor. This is because the average signal is averaged over the fading effect so the remaining factors are the transmission power and path loss, being identical to the location based scheme. We formally state this in the following lemma.

Lemma 5. *The average signal based user association scheme is equivalent to the location based scheme with* $\tau_i = \kappa_i^\alpha / P_{t,i}$, $\forall i$.

Proof: See Appendix F. □

Theorem 5 ([29])**.** *The coverage probabilities for the instantaneous SINR based scheme are*

$$\mathbb{P}_{\text{INS}} = \sum_{i=1}^{K} \lambda_i \int_{r=0}^{\infty} 2\pi r \exp(-(\sum_k \lambda_k P_{t,k}^{2/\alpha})C(\alpha) (\gamma_i/P_{t,i})^{2/\alpha} r^2) \exp(-(\gamma_i/P_{t,i})\sigma^2 r^\alpha) \mathrm{d}r \quad (17)$$

$$\mathbb{P}_{\text{INS},\sigma^2=0} = \frac{\pi}{C(\alpha)} \frac{\sum_{i=1}^{K} \lambda_i P_i^{2/\alpha} \gamma_i^{-2/\alpha}}{\sum_{i=1}^{K} \lambda_i P_i^{2/\alpha}} \quad (18)$$

where $C(\alpha) = \frac{2\pi^2}{\alpha} \csc(2\pi/\alpha)$.

A. Constrained Optimization Framework

Similar to the previous section, we investigate the problem of minimizing energy consumption subject to a QoS constraint in terms of coverage probability.

1) Power Consumption Minimization with Average Signal based Scheme: In the following, we formulate an optimization problem that minimizes energy consumption across different tiers. Using Theorem 4 and Lemma 5, we obtain the following corollary.

Corollary 1. *If we connect to the highest average SINR signal, the coverage probabilities are given by*

$$\mathbb{P}_{\text{SIG}} = \sum_i \lambda_i P_{t,i}^{2/\alpha} \int_{r=0}^{\infty} 2\pi r \exp(-r^\alpha \gamma \sigma^2) \exp(-\pi r^2 (\sum_i \lambda_i P_{t,i}^{2/\alpha})(1 + \rho(\gamma, \alpha))) \mathrm{d}r \quad (19)$$

$$\mathbb{P}_{\text{SIG},\sigma^2=0} = \frac{1}{1 + \rho(\gamma, \alpha)} \quad (20)$$

Proof: Let $\kappa_i = P_{t,i}^{1/\alpha}$ in Theorem 4. The result is obtained after some algebraic manipulations. □

We investigate now the following optimization problem:

$$\mathcal{P}_{\text{SIG}} : \begin{cases} \min\limits_{\lambda_i, \forall i} & \sum_i \lambda_i (P_{i0} + P_i) \\ \text{s.t.} & \mathbb{P}_{\text{OAP}} \geq \epsilon \end{cases} \quad (21)$$

For our analysis, it is necessary to consider the cases $\sigma^2 = 0$ and $\sigma^2 > 0$ separately. When $\sigma^2 = 0$, the solution is to choose λ_i as small as possible, for all i. Hence, when the network is dense, it is beneficial to shut down as many access points as possible. However, this observation is no longer valid when the network is sparse as the assumption $\sigma^2 = 0$ is no longer valid. Now, suppose $\sigma^2 > 0$, we denote $S = \sum_i \lambda_i P_{t,i}^{2/\alpha}$ for notational convenience. As a consequence of Lemma 3, the optimal $S^\star = \sum_i \lambda_i^\star P_{t,i}^{2/\alpha}$ satisfies

$$\epsilon = S^\star \int_{r=0}^{\infty} 2\pi r \exp(-r^2 \pi S^\star (1 + \rho(\gamma, \alpha))) \exp(-r^\alpha \gamma \sigma^2) \mathrm{d}r. \quad (22)$$

This reduces the original minimization problem in (22) to

$$\mathcal{P}_{\text{SIG}}^0 : \begin{cases} \min\limits_{\lambda_i, \forall i} & \sum_i \lambda_i (P_{i0} + P_i) \\ \text{s.t.} & \sum_i \lambda_i P_{t,i}^{2/\alpha} = S^\star \end{cases} \quad (23)$$

which is a linear program having as solution the tier that minimizes $(P_{i0} + P_i) P_{t,i}^{-2/\alpha}$. This minimization problem can be further adapted to include certain constraints on λ_i and it still gives a linear program (for example, the macro tier structure

TABLE I
PARAMETER VALUES USED IN NUMERICAL SECTION.

Parameter	Value
α	4
λ, μ	10^{-4}m^{-2},10^{-3}m^{-2}
$P_{t,\text{M}}, P_{t,\text{F}}$	43 dBm, 10 dBm
σ^2	1
γ	-10 dB
P_{sleep}	75.0 W (Macro only)
$P_{\text{M0}}, P_{\text{F0}}$	130.0 W, 4.8 W
$\Delta_{\text{M}}, \Delta_{\text{F}}$	4.7, 8.0
$P_{\text{M}}, P_{\text{F}}$	20.0 W, 0.05 W

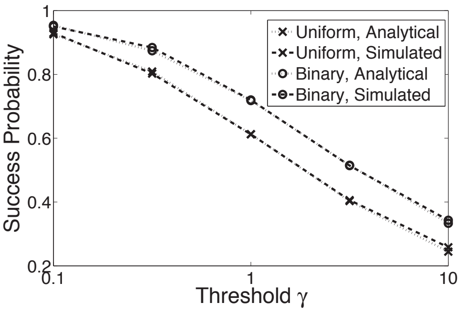

Fig. 1. Comparison of analytical expressions vs. simulated results for strategic sleeping mode.

is an existing infrastructure and this could be reflected by fixing λ_{M}). In a more general setting, one could include β_i representing power control as a decision variable (replace $P_{t,i}$ with $\beta_i P_{t,i}$) though the resulting minimization problem would require numerical computation.

2) Energy Efficiency Optimization with Instantaneous SINR based Scheme: In the following, we shall consider that the network has two tiers, a macro tier where random sleeping is implemented and a femto tier that does not implement any sleeping strategy. Given the density of the femtocell access points λ_F, we want to determine the value of $q^\star_{\text{INS}}$ that optimizes the energy efficiency. Since the equations are intractable in general, we assume that $\sigma^2 = 0$ as a means to obtain some insight. The problem formulation is given by

$$\mathcal{P}_{\text{INS}} : \begin{cases} \max\limits_{q} & \frac{\pi}{C(\alpha)} \frac{q\lambda_{\text{M}} P_{t,\text{M}}^{2/\alpha}\gamma^{-2/\alpha} + \lambda_{\text{F}} P_{t,\text{F}}^{2/\alpha}\gamma^{-2/\alpha}}{q\lambda_{\text{M}} P_{t,\text{M}}^{2/\alpha} + \lambda_{\text{F}} P_{t,\text{F}}^{2/\alpha}} \\ & \times \frac{\log_2(1+\gamma)(\lambda_{\text{M}} q + \lambda_{\text{F}})}{\lambda_{\text{M}}(qP_{\text{M0}} + q\Delta_{\text{M}} P_{\text{M}} + (1-q)P_{\text{sleep}}) + \lambda_{\text{F}}(P_{\text{F0}} + \Delta_{\text{F}} P_{\text{F}})} \end{cases}$$

which is monotone decreasing in q and hence has optimal $q^\star_{\text{INS}} = 0$.

V. NUMERICAL RESULTS

In the following, we use the default values in Table I unless otherwise stated. The parameters concerning the power consumption are obtained from [6].

We shall consider two models of activity levels: *binary* where the activity level associated with each coverage area, which in turn is associated with a particular MBS is either 0 or 1 with probability 0.5 each, and *uniform* where the activity level is drawn from a uniform $[0, 1]$ random variable. The sleeping strategy for both cases is identical: if the activity level in the coverage area associated with the MBS is a, then the MBS stays awake with probability a. We also calculate the coverage probability through Monte Carlo simulation. The locations of the MBSs are distributed according to a PPP in a $5000\text{m} \times 5000\text{m}$ grid, with 5000 trials. Fig. 1 compares the analytical results versus the simulated results, verifying the validity of the expression (9) concerning the strategic sleeping strategy for $\sigma^2 = 0$. From henceforth, all figures are numerical plots of the expressions obtained previously.

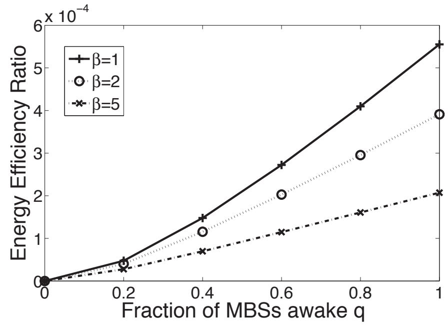

Fig. 2. Effect of power control on energy efficiency.

Fig. 2 shows the energy efficiency with random sleeping with respect to q for various values of β (expression (6) divided by expression (4)). From this figure, we observe that the energy efficiency increases with q. This is because the network throughput decreases at a faster rate than the savings in power consumption when we decrease q. The figure also shows that the energy efficiency decreases with increasing β, which implies that the cost incurred from raising the power uniformly is not compensated by an increase in the data rate. Note that this result has not yet taken into account traffic demands and different operating power consumption parameters at the MBS. Therefore, it is likely that taking into account these additional parameters will give us new tradeoffs, which will be studied in future work. Nevertheless, our framework does give a simple tractable approach to study the effect of random sleeping in macrocell networks.

Fig. 3 plots the coverage probability versus noise σ^2 for different sleeping strategies (eq. (8)) while Fig. 4 plots the energy efficiency with respect to q for various sleeping strategies (eq. (8) divided by eq. (5)). For Fig. 3, the activity model for strategic sleeping is assumed to be 0 and 1 with equal probability 0.5. The sleeping strategy is modeled as 0 and 1, respectively. For random sleeping, MBSs are in sleep mode with probability 0.5. From the plots, we can see that the coverage probability per active user in strategic sleeping is only marginally better than no sleeping. We also see that strategic sleeping has a bigger margin of improvement over no sleeping when $\sigma^2 \to 0$. In this figure, we see that even for a contrived example, there is little improvement when noise is significant. On the other hand, our analytical results demonstrate that when $\sigma^2 = 0$, any increasing strategy $S(a)$ would suffice. This implies that the presence of noise can significantly affect the performance. Finally, it can be seen that expectedly, strategic sleeping is always better than random sleeping for the same fraction of sleeping MBSs. In Fig. 4, we choose the strategic sleeping model to have a activity 1 with probability q, represented by the $x-$axis, and activity 0 otherwise. Likewise the sleeping strategy is 1 if the activity is

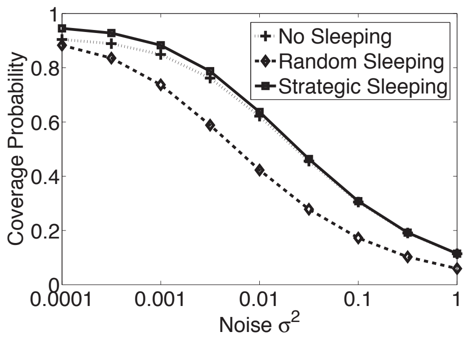

Fig. 3. Coverage probabilities for different sleeping strategies.

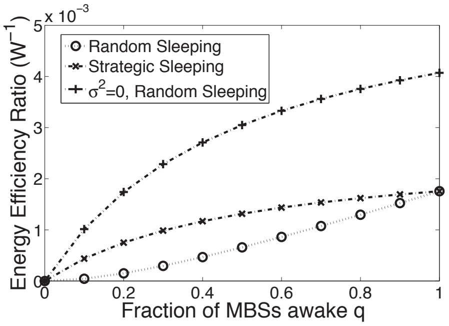

Fig. 4. Energy efficiency ratio for different sleeping strategies.

1, 0 otherwise. To obtain a fair comparison, we also plot the random sleeping with MBS staying awake with probability q so that both plots have the same fraction of active MBSs. From Fig 4, we observe that the energy efficiency for a strategic sleeping strategy is also higher than random sleeping and in fact, for these set of parameters, is about half of the interference-limited regime case for all values of q.

VI. Conclusion

In this paper, we investigated the design of energy efficient cellular networks through the employment of base station sleep mode strategies as well as small cells, and investigated the tradeoff issues associated with these techniques. Using a stochastic geometry based model, we derived the success probability and energy efficiency under sleeping strategies in homogeneous macrocell and heterogeneous K-tier networks. In addition, we formulated optimization problems in the form of power consumption minimization and energy efficiency maximization and determined the optimal operating frequency of the macrocell base station. In particular, we investigated the impact of random sleeping and strategic sleeping on the power consumption and energy efficiency. Numerical results confirmed the effectiveness of sleeping strategy in homogeneous macrocell networks but the gain in energy efficiency depends on the type of sleeping strategy used. In addition, the deployment of small cells generally leads to higher energy efficiency but this gain saturates as the density of small cells increases.

Future work may include the extension of the above model to the case where base stations have multiple antennas and may perform opportunistic user selection. It would also be of interest to explore how random spatial placements of base stations that model repulsion or inhibition affect the results in terms of throughput and energy efficiency. Finally, the energy efficiency metric investigated here is only dependent on the power consumption and the coverage within the network, and does not take into account the infrastructure cost and backhaul overhead associated with implementing small cell networks.

VII. Appendix

A. Useful statistics concerning success probabilities

One very useful consequence of the Rayleigh fading model and the definition of the SINR is that, for a fixed distance from the transmitter, the probability of a successful transmission can expressed as a product of Laplace transforms of independent random variables. The details of the exact derivation can be found in many articles that approach modeling wireless networks with stochastic geometry, such as [16]. Here, instead of duplicating the calculations and proofs, we just state the known, standard results. The statistics concern the probability of a successful transmission from the BS to the user (downlink), given that the distance separating both the BS and user is r.

The success probability given that the interferers are distributed as a PPP, transmitting at the same transmit power as the BS that is assigned to the user, and are located at a distance at least r from the user, is given by:

$$\mathcal{L}_I(r) = \exp(-\pi r^2 \lambda_{\mathrm{M}} \gamma^{2/\alpha} \rho(\gamma, \alpha)). \tag{24}$$

Notice that in our model, we assume that each user is assigned to the nearest MBS and hence, all other MBS (which are now interferers) must be at least r from the user.

The success probability given that the background noise is σ^2 is given by:

$$\mathcal{L}_N(r) = \exp(-r^\alpha \gamma \sigma^2 / P_{t,M}). \tag{25}$$

Suppose one wants to compute the success probability in the first scenario and in addition to that, account for background noise σ^2. As a consequence of the Laplace transform formulations, the desired success probability turns out to be the product of both expressions

$$\mathcal{L}_{I,N}(r) = \exp(-\pi r^2 \lambda_{\mathrm{M}} \gamma^{2/\alpha} \rho(\gamma, \alpha)) \exp(-r^\alpha \gamma \sigma^2 / P_{t,M}). \tag{26}$$

B. Proof of Theorem 1

The coverage probability is defined as

$$\int_{r=0}^{\infty} \mathcal{L}_I(r) \mathcal{L}_N(r) f_{\lambda_{\mathrm{M}}}(r) \mathrm{d}r \tag{27}$$

where the probability density function of the MBS $f_{\lambda_{\mathrm{M}}}(r)$ is $2\pi\lambda_{\mathrm{M}} r \exp(-\pi\lambda_{\mathrm{M}} r^2)$ (without sleeping) and $2\pi q \lambda_{\mathrm{M}} r \exp(-\pi q \lambda_{\mathrm{M}} r^2)$ (with sleeping).

C. Proof of Theorem 2

The first step is to condition on the activity of a typical cell $a(x)$. Next, we enumerate all the MBSs in increasing order of distance from the user, starting from 1.[3] $N_{\rm ord}$ denotes the order of the MBS the user connects to and $f_A(x)$ denotes the pdf of A. The success probability per link is thus given by

$$\begin{aligned}\mathbb{P}_{\rm SS} &\stackrel{(a)}{=} \frac{1}{\mathbb{E}\{a\}}\int_0^1 x\mathbb{P}(\mathsf{SINR}>\gamma|x)f_A(x){\rm d}x\\ &\stackrel{(b)}{=} \frac{1}{\mathbb{E}\{a\}}\int_0^1 \{x\mathbb{P}(N_{\rm ord}=1)\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}=1)\\ &\quad +x\mathbb{P}(N_{\rm ord}>1)\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}>1)\}f_A(x){\rm d}x\\ &\stackrel{(c)}{=} \frac{1}{\mathbb{E}\{a\}}\int_0^1 \Big\{xs(x)\int_{r=0}^{\infty}\exp(-\pi r^2\lambda_{\rm M}\mathbb{E}\{s\}\rho)\\ &\quad \exp(-\pi r^2\lambda_{\rm M})\exp(-r^\alpha\gamma\sigma^2/P_{t,{\rm M}}){\rm d}r\\ &\quad +x(1-s(x))\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}>1)\Big\}f_A(x){\rm d}x \quad (28)\end{aligned}$$

where (a) is by definition of a coverage probability weighted over the active user links, (b) partitions into the event of the nearest MBS being awake and the event of the nearest MBS being asleep, and (c) is from the Laplace transform of the remaining active interferers, distributed as a PPP with intensity $\mathbb{E}\{s\}\lambda_{\rm M}$, and the pdf of the nearest MBS. This leads us to $\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}>1)$, which is given by

$$\begin{aligned}&\mathbb{P}_{\rm SS}(\rightarrow|N_{\rm ord}>1)\\ &\stackrel{(a)}{=} \sum_{i=2}^{\infty}\mathbb{P}(N_{\rm ord}=i|N_{\rm ord}>1)\mathbb{P}(\rightarrow|N_{\rm ord}=i)\\ &\stackrel{(b)}{=} \sum_{i=2}^{\infty}\mathbb{E}\{s\}(1-\mathbb{E}\{s\})^{i-2}\int_{r=0}^{\infty}\exp\Big(-\frac{r^\alpha\gamma\sigma^2}{P_{t,{\rm M}}}\Big)\\ &\quad \exp(-\pi r^2\lambda_{\rm M}\mathbb{E}\{s\}(1+\rho))2(\lambda_{\rm M}\pi)^i r^{2i-1}{\rm d}r \quad (29)\end{aligned}$$

where (a) splits into the events "connect to the $i-$th MBS", (b) is the Laplace transform of the interference term and the pdf of the i-th MBS.

In the case where $\sigma^2=0$, the term $\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}>1)$ simplifies to $\sum_{i=2}^{\infty}\frac{\mathbb{E}\{s\}(1-\mathbb{E}\{s\})^{i-2}}{(1+\mathbb{E}\{s\}\rho)^i}=\frac{1}{1+\rho}\frac{1}{1+\mathbb{E}\{s\}\rho}$, which, combined with (28), leads to (9).

D. Proof of Lemma 4

Once again, we use the notation $N_{\rm ord}$ to denote the order of the MBS that the user is connected to. In addition, we impose that all strategies are measurable functions. Firstly, note that $\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}=1)$ is more than $\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}>1)$ or in other words, the success probability when the nearest active MBS is the nearest MBS than if not. This is intuitively evident. Next, note that the optimal $S^\star(a)$ is completely characterized by $a^\star$. Now, suppose we have a strategy $S_1(a)$ that is not (almost surely) $S^\star(a)$. Then, by definition of being different, there exists a $\epsilon>0$ such that the set $B=\{x, x\geq a^\star, S_1(x)<1-\epsilon\}$ has measure >0. Roughly speaking, we just find a set where the strategy is not 1. Next, we construct another strategy $S_2(a)$ from $S_1(a)$ while retaining $\mathbb{E}\{s\}$. Roughly, we part from the function $S_1(x)$ for $x<a^\star$ and $S_1(x)>0$ and "fill" up the set B. Then, using $\mathbb{P}(\mathsf{SINR}>\gamma|N_{\rm ord}=1)$ and by the notion that "we serve more people by switching on in areas with higher activity than in areas with lesser activity", we arrive at the conclusion that S_2 has a higher coverage rate per active user than S_1. Lastly, the only function where we cannot do this sort of procedure is precisely the one characterized by $S^\star$.

[3]The distance of each MBS from the user is almost surely distinct.

E. Proof of Theorem 4

Let $f_i(r)=2\pi\lambda_i r\exp(-\pi\lambda_i r^2)$ denotes the pdf of the distance to the nearest BS in tier i. First, we compute the probability of connecting to tier i, i.e. $\mathbb{P}(\kappa_i r_i<\kappa_j r_j\forall j\neq i)$ as follows:

$$\begin{aligned}&\mathbb{P}(\kappa_i r_i<\kappa_j r_j\forall j\neq i)\\ &=\int_{r=0}^{\infty}f_i(r)\mathbb{P}(\kappa_i r<\kappa_j r_j\forall j\neq i){\rm d}r\\ &=\int_{r=0}^{\infty}f_i(r)\big(\prod_{j\neq i}\int_{r_j=r\kappa_i/\kappa_j}^{\infty}f_j(r_j){\rm d}r_j\big){\rm d}r\\ &=\int_{r=0}^{\infty}f_i(r)\big(\prod_{j\neq i}\exp(-\pi\lambda_i(r\kappa_i/\kappa_j)^2)\big){\rm d}r\\ &=\frac{\lambda_i\kappa_i^2}{\sum_j\lambda_j\kappa_j^2}. \quad (30)\end{aligned}$$

Now, conditioned on the event that the user is connected to the i-th tier, we derive the probability of a successful transmission. This requires us to determine the Laplace Transform of the interference and noise terms. For the Laplace Transform of the noise term, it is given in (25). As such, we need to derive the generic Laplace transform due to interference I from transmitters from a general tier j (including i) [16], [18]:

$$\begin{aligned}\mathcal{L}_{I(j)}(s)&=\mathbb{E}_{\Theta_j}\left[\exp_{\mathcal{H}}[\exp(-sh_yP_{t,i}x_y^{-\alpha})]\right]\\ &=\mathbb{E}_{\Theta_j}\left[1/(1+sP_{t,i}x_y^{-\alpha})\right]\\ &=\exp\left(-2\pi\lambda_i\int_{r\kappa_i/\kappa_j}^{\infty}\left(1-\frac{1}{1+sv^{-\alpha}}\right)vdv\right)\end{aligned} \quad (31)$$

where the last step follows from known results about the probability generating functional (PGFL) of PPPs. Following the definition of the success probability as $\mathbb{P}(\mathsf{SINR}>\gamma)$, we compute $\mathbb{E}_h[P_{t,i}hr^{-\alpha}>\gamma I(j)]$ and after some algebraic manipulations, we get $\mathcal{L}_{I(j)}(\gamma r^\alpha/P_{t,i})=\exp\left(-2\pi\lambda_i\int_{r\kappa_i/\kappa_j}^{\infty}\left(1-\frac{1}{1+\gamma r^\alpha/P_{t,i}v^{-\alpha}}\right)vdv\right)=\exp(-\pi(r\kappa_i/\kappa_j)^2\lambda_i\rho(\gamma P_{t,i}\kappa_j^\alpha/P_{t,j}\kappa_i^\alpha,\alpha))$.
The success probability is given by $\sum_i\frac{\lambda_i\kappa_i^2}{\sum_j\lambda_j\kappa_j^2}\int_{r_i=0}^{\infty}\Big(\prod_j\mathcal{L}_{I(j)}(\gamma r^\alpha/P_{t,i})\Big)\mathcal{L}_N f_i(r){\rm d}r$ and so the final step is to combine the previously obtained expressions and integrate w.r.t. r.

F. Proof of Lemma 5

First note that if we have two transmitters of the same type placed at distance x and y away where $x<y$, then the average signal from x is smaller than the average signal from y. Hence, we can assume that we always connect to the nearest BS from each tier i. To prove this result, we need to verify that, suppose $\tau_x=\kappa_x^\alpha/P_{t,x}$ for all x holds, whenever we have two tiers satisfying the relation $r_i\kappa_i=r_j\kappa_j$, then the relation $Q_i\tau_i=Q_j\tau_j$ holds as well (Q is the perceived average SINR signal).

For a user connecting to tier i, the SINR is given by

$$\mathsf{SINR} = \frac{P_{t,i,1} h_{i,1} r_{i,1}^{-\alpha}}{\sigma^2 + P_A + P_B + P_C + P_D} \tag{32}$$

where $P_A = P_{t,j,1} h_{j,1} r_{j,1}^{-\alpha}$, $P_B = \sum_{k\geq 2} P_{t,i,k} h_{i,k} r_{i,k}^{-\alpha}$, $P_C = \sum_{k\geq 2} P_{t,j,k} h_{j,k} r_{j,k}^{-\alpha}$, $P_D = \sum_{x\neq\{i,j\},k\geq 1} P_{t,x,k} h_{x,k} r_{x,k}^{-\alpha}$. As part of conditional probability, the term representing interference from tier i (and j also), $\sum_{k\geq 2} P_{t,i,k} h_{i,k} r_{i,k}^{-\alpha}$, is conditioned on that the transmitters index ≥ 2 are at r_i and beyond, while for tier $x \neq \{i,j\}$, all the transmitters are at r_x and beyond. By Slivnyak's Theorem, the transmitters are also distributed as homogeneous PPPs. Recalling the steps in (31) (and also, $\mathbb{E}_h[\exp(-P_{t,i} r_i^{-\alpha} h_i)] = 1/(1 + P_{t,i} r_i^{-\alpha})$), we get the Laplace transform, hence success probability, for connecting to tier i as

$$\mathcal{L}_i(\gamma r_i^\alpha / P_{t,i}) = \frac{\exp(-\gamma r_i^\alpha \sigma^2 / P_{t,i})}{1 + P_{t,j} r_j^{-\alpha}} \times \left\{ \prod_k \exp\left(-\pi r_i^2 \kappa_k^2 \kappa_i^{-2} \lambda_k \rho\left(\frac{\gamma P_{t,k} \kappa_i^\alpha}{P_{t,i} \kappa_k^\alpha}, \alpha \right) \right) \right\}. \tag{33}$$

Therefore, the relation $Q_i \tau_i = Q_j \tau_j$ is equivalent to the relation $\mathbb{P}_i(\tau_i \mathsf{SINR} > \gamma) = \mathbb{P}_j(\tau_j \mathsf{SINR} > \gamma)$ for all γ. Define $\gamma_i = \gamma/\tau_i$. To verify the relation $Q_i \tau_i = Q_j \tau_j$, replace γ with γ_i in (33). Continue by plugging in $r_i \kappa_i = r_j \kappa_j$ together with $\tau_x = \kappa_x^\alpha / P_{t,x}$ for all x and perform a series of algebraic manipulations to verify that the two success probabilities are indeed equal. This verifies that the relation $Q_i \tau_i = Q_j \tau_j$ also holds at the same time. Thus, the two schemes are equivalent with the relation $\tau_x = \kappa_x^\alpha / P_{t,x}$ for all x.

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