Refunding for Small Cell Networks with Limited-Capacity Backhaul

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Abstract—**Small cells can offload macrocell traffic, improve indoor coverage and cell-edge user performance, and boost network capacity. However, backhaul is one of the key constraints for future small cell networks. In this paper, we investigate the refunding mechanism for small cell networks with limited-capacity backhaul, in which small cell holders (SHs) can serve guest users (GUs) with their remaining backhaul capacity. In return, SHs can receive refunding from mobile network operator (MNO) as incentives. Specifically, we formulate this problem as a Stackelberg game with MNO being a leader and SHs being followers. The MNO decides individualized refunding and interference temperature constraints to different SHs. Subsequently, each SH serves GUs in a best-effort manner while maximizing its utility function in terms of refunding and logarithmic throughput. To reach subgame perfect equilibrium, we propose a novel look-up table approach at MNO and an optimal power allocation algorithm at SHs through majorization theory.**

I. Introduction

Small cells are low power access points operated in licensed spectrum, which encompass femtocells, picocells, microcells, and metrocells. Nowadays, small cells are increasingly installed in offices, subways, and residential sites to cope with the exponential growth of mobile traffic and the need for ubiquitous mobile access [1-2]. In fact, backhaul communication is arguably the key challenge for small cells [3]. Currently, the majority of backhaul are wired links, e.g., 70%-80% in U.S. and 40% worldwide in 2010. It is predicted that mobile networks will require a 10x fatter backhaul in 2016. In literature, the backhaul limitation usually appears as constraints to the sum-rate or transmission delay [4-5] . On the other hand, the existing refunding frameworks on femtocells assume unlimited backhaul capacity, which is infeasible in practical implementation [6-7].

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In this paper, we investigate the refunding and optimization for small cell networks with limited-capacity backhaul. For a specific small cell, we refer the registered users as home users (HUs) and others as guest users (GUs). Due to selfish nature, SHs prefer to reserve backhaul capacity for their own use rather than sharing with GUs, thus resulting in unused backhaul capacity. Hence, the MNO would like to motivate SHs to serve GUs with their remaining backhaul capacity if possible. In return, SHs receive refunding from MNO as incentives. We consider best-effort service provisioning is applied for GUs. We use a similar majorization theory method to obtain the best response function for each SH [8]. Instead of maximizing sum-rate, the key difference is that the objective of our problem is a tradeoff between refunding in terms of GUs' achievable sum-rate and HU's utility degradation.

The key contributions of this paper are summarized as follows:

- *Novel Stackelberg Game Formulation*: We provide a novel Stackelberg game formulation combining economical refunding and technical specifications. In Stage I, the MNO maximizes net revenue subject to the aggregate interference constraint. In Stage II, each SH maximizes its utility subject to power, interference temperature, and backhaul capacity constraints.
- *Refunding with Best-Effort Service Provisioning*: The utility of SH is defined as the sum of refunding in terms of GUs' sum-rate and HUs' logarithmic throughputs. We obtain the optimal transmission powers for GUs through majorization theory.
- *Individualized Refunding*: We propose a look-up table approach to solve the revenue maximization problem in Stage I. The MNO decides individualized refunding and interference temperature constraints to different SHs.

II. System Model and Problem Formulation

We consider an uplink heterogeneous network, which consists of one macrocell operated by MNO overlaided

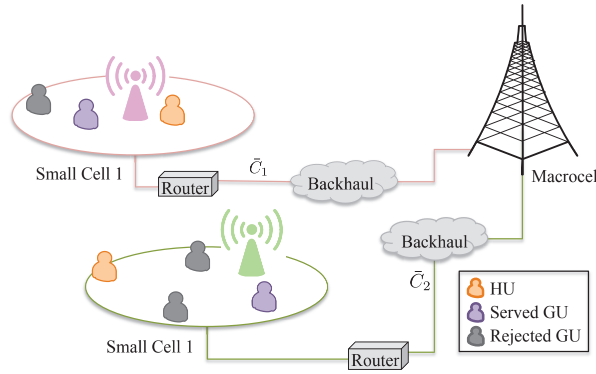

Figure 1. System Model

with M small cells installed by SHs indexed as $\mathcal{M} = \{1, 2, \cdots, M\}$ in Fig. 1. All the small cells are connected to the macrocell through wired backhaul links. The MNO refunds SHs for their sharing of backhaul capacity with GUs. The macrocell and small cells operate in two separate frequency bands so there is no cross-tier interference. Small cells are densely deployed and are sharing the same frequency band, resulting in intra-cell interference. In the following, we assume the mth ($m \in \mathcal{M}$) small cell has *one* HU and N_m GUs at each time slot. The HU is indexed as 0 and GUs are indexed as $\mathcal{N}_m = \{1, 2, \cdots, N_m\}$. Note that each GU belongs to one small cell at one time slot. The approximated SINR of HU in the mth small cell is given by

$$\mathrm{SINR}_0^m (\mathbf{p}^m) = \frac{p_0^m h_0^m}{\sum_{i=1}^{N_m} p_i^m h_i^m + n_m} \tag{1}$$

where h_i^m denotes the fading gain from the ith user to its associated the mth small cell; p_0^m is the HU's fixed transmission power and $\mathbf{p}^m = \left[p_1^m, p_2^m, \cdots, p_{N_m}^m\right]$ are the GUs' transmission powers. We assume that the interference generated by the mth small cell to the neighboring small cells is bounded by I_m. Hence, the aggregated interference to the mth small cell from neighboring small cells is approximated as $I_{-m} = \sum_{n \neq m} I_n$. The aggregated noise power becomes $n_m = \sigma_m^2 + I_{-m}$, where σ_m^2 is the additive white Gaussian noise power.

Similarly, the approximated SINR of the ith GU in the mth small cell is given by

$$\mathrm{SINR}_i^m (\mathbf{p}^m) = \frac{p_i^m h_i^m}{\sum_{j=1, j \neq i}^{N_m} p_j^m h_j^m + n'_m}, \forall i \in \mathcal{N}_m \tag{2}$$

where $n'_m = p_0^m h_0^m + n_m$. In the following, the fading gain is modeled as exponential power fading gain which corresponds to Rayleigh fading channel. The fading channels are initially estimated between users and small cells and these channel gains are made available to the SHs. With Rayleigh fading assumption, h_i^m is simply an independent exponentially random variable with unit mean. Hence, the outage probability of HU is given by [9]

$$\begin{aligned} \mathrm{P}_{\mathrm{out}}^m (\mathbf{p}^m) &= \Pr\left(\mathrm{SINR}_0^m (\mathbf{p}^m) \leq \gamma_0^m\right) \\ &= 1 - e^{\left(-\frac{\gamma_0^m n_m}{p_o^m}\right)} \prod_{i=1}^{N_m} \left(1 + \frac{\gamma_0^m p_i^m}{p_o^m}\right)^{-1} \end{aligned} \tag{3}$$

where $\gamma_0^m \geq 1$ is the fixed target SINR of HU. One QoS metric of HU is the throughput, which is a function of outage probability and, in turn, depends on the transmission powers $\mathbf{p}^m$. Specifically, the HU's throughput is given by

$$c_m = \left[1 - \mathrm{P}_{\mathrm{out}}^m (\mathbf{p}^m)\right] \log\left(1 + \gamma_0^m\right) \tag{4}$$

and the logarithmic utility of HU is then defined as [10]

$$T_m = \log(c_m) = -\sum_{i=1}^{N_m} \log\left(1 + \frac{\gamma_0^m p_i^m}{p_0^m}\right) + A_m \tag{5}$$

where $A_m = -\frac{\gamma_0^m n_m}{p_0^m} + \log\left[\log\left(1 + \gamma_0^m\right)\right]$ is a constant. It is apparent that each GU contributes to the HU's utility degradation independently, i.e., if the ith GU is rejected by the SH, the corresponding term $-\log\left(1 + \frac{\gamma_0^m p_i^m}{p_0^m}\right)$ vanishes without affecting HU's utility.

For small cells with best-effort service provisioning, the refunding function Φ_m is defined in terms of GU's sum-rate as

$$\Phi_m = \phi_m R_m (\mathbf{p}^m), \; m \in \mathcal{M} \tag{6}$$

where ϕ_m is the unit refunding per time slot and the GU's sum-rate $R_m(\mathbf{p}_m)$ is given by

$$R_m (\mathbf{p}^m) = \sum_{i=1}^{N_m} \log\left(1 + \frac{h_i^m p_i^m}{\sum_{j \neq i} h_j^m p_j^m + n'_m}\right). \tag{7}$$

The mth SH's utility is defined as the sum of refunding from the MNO and HU's logarithmic throughput and it is given by

$$U_m = \Phi_m + \nu T_m, \; m \in \mathcal{M} \tag{8}$$

where ν is the unit convert between monetary utility Φ_m and performance utility T_m. Serving GUs can receive refunding from the MNO but, as a consequence, cause HU's utility degradation. However, rejecting GUs can secure a better QoS for HU with the sacrifice of refunding. It is important for SHs to balance between refunding and performance, i.e., determining $R_m(\mathbf{p}_m)$ that maximizes (8).

Finally, the MNO's net revenue is the total service gain from GUs minus total refunding to SHs and it is given by

$$U_{\text{MNO}} = \sum_{m=1}^{M} \pi \min\left\{\bar{C}_m,\ R_m\left(\mathbf{p}^m\right)\right\} - \sum_{m=1}^{M} \phi_m R_m\left(\mathbf{p}^m\right) \tag{9}$$

where π is the usage-based fee per time slot and $\bar{C}_m$ is the remaining backhaul capacity after serving the HU. The GUs only need to pay to the MNO when they are served by SHs.

We formulate a Stackelberg game to analyze the interactions between the MNO and SHs, where the MNO acts as a leader and M SHs as followers. In Stage I, the MNO first announces individualized refunding and interference temperature constraints to different SHs. In Stage II, the SHs compete in a non-cooperative manner and adjust their corresponding strategies.

In Stage I, the optimization problem at the MNO is formulated as

$$\mathcal{P}_{\text{L}} := \begin{cases} \underset{\{I_m\},\{\phi_m\}}{\text{maximize}} & U_{\text{MNO}} \\ \text{subject to} & \sum_{m=1}^{M} I_m \le Q \\ & I_m \ge g_m p_0^m \\ & 0 \le \phi_m \le \pi \end{cases} \tag{10}$$

where Q is the upper bound of the aggregated interference generated by all small cells, $g_m = \max\limits_{k \neq m}\left(d_{mk}^{-\alpha}\right)$, where d_{mk} is the distance between the mth and the kth small cells, and α is the path loss coefficient. The objective function of the problem $\mathcal{P}_{\text{L}}$ is the MNO's net revenue. It consists of two terms: the first term is the total service gain, where the MNO can, at most, charge $\pi \min\left\{\bar{C}_m,\ R_m\left(\mathbf{p}^m\right)\right\}$ from GUs in the mth small cell and the second term is the total refunding to SHs.

In Stage II, the optimization problem at each small cell is formulated as

$$\mathcal{P}_{\text{F}}^m := \begin{cases} \underset{\mathbf{p}^m}{\text{maximize}} & U_m \\ \text{subject to} & 0 \le p_i^m \le p^{\text{MAX}} \\ & \sum_{i=1}^{N_m} p_i^m \le \frac{I_m}{g_m} - p_0^m \end{cases} \tag{11}$$

where p^{MAX} is the GU's maximum transmission power. We model the aggregated interference generated by the mth small cell approximately as $g_m\left(\sum_{i \in \Omega_m} p_i^m + p_0^m\right)$.

Lemma 1. *In Stage I, the optimal sum-rate $R_m^\star$ is always less than or equal to $\bar{C}_m$, $\forall m \in \mathcal{M}$.*

Proof: By contradiction, if $R_m^\star > \bar{C}_m$ for some m, we can always find $R_m^{'} = \bar{C}_m$ such that $U_{\text{MNO}}^{'} > U_{\text{MNO}}^{\star}$. Hence, the term $\min\left\{\bar{C}_m,\ R_m\left(\boldsymbol{p}_m\right)\right\}$ in the problem $\mathcal{P}_{\text{L}}$ is equivalent to insert a backhaul constraint $R_m\left(\mathbf{p}^m\right) \le \bar{C}_m$ to the problem $\mathcal{P}_{\text{F}}^m$. □

III. Backward Induction Method

In this section, we use backward induction to solve the Stackelberg game and obtain the subgame perfect equilibrium. The Stage II problem is solved first for a given I_m and ϕ_m. Based on the best response function from each small cell, the MNO maximizes the net revenue by determining individualized refunding and interference temperature constraints to different SHs.

A. Optimal Power Allocation

Now, we focus on solving $\mathcal{P}_{\text{F}}^m$.

Lemma 2. *Denote $x_i^m = h_i^m p_i^m$, the GUs' sum-rate (7) is a strictly schur-convex function on the feasible convex domain $\mathcal{D}_m = \left\{\mathbf{x}^m | 0 \le x_i^m \le h_i^m p^{MAX},\ \sum_{i=1}^{N_m} \frac{x_i^m}{h_i^m} \le \frac{I_m}{g_m} - p_0^m\right\}$.*

Proof: Fix $\sum_{i=1}^{N_m} x_i^m = X^m$, we can write $R_m(\mathbf{x}^m)$ as $\sum_{i=1}^{N_m} \log\left(\frac{X^m + n_m'}{X^m - x_i^m + n_m'}\right)$. It is easy to prove that $R_m(\mathbf{x}^m) = \sum_{i=1}^{N_m} \log\left(\frac{X^m + n_m'}{X^m - x_i^m + n_m'}\right)$ is a separable convex function on $[0,\ X^m]^{N_m}$, which implies that $R_m(\mathbf{x}^m)$ is a strictly schur-convex function on $\mathcal{D}_m$. □

Based on the Schur-convexity of sum-rate (7) and the concavity of logarithmic utility, i.e., $\sum_{i=1}^{N_m} \log\left(1 + \frac{\gamma_0^m p_i^m}{p_0^m}\right)$, we have the following theorem to obtain the GU's optimal transmission powers for a given ϕ_m and I_m.

Theorem 3. *At the optimal solution of $\mathcal{P}_F^m$,*

1) *if more than two GUs can transmit, it must satisfy:*
   a) *there is at most one GU transmitting at 0 or $\min\left\{p^{MAX},\ \frac{I_m}{g_m} - p_0^m - l p^{MAX}\right\}$, where l is the number of GUs transmitting at full power;*
   b) *the other GUs transmit either at full or zero power;*
   c) *the transmitted GUs have better channel gains than non-transmitted ones.*
2) *If only one GU transmits, it must satisfy:*
   a) *the transmitted GU has the best channel gain among all GUs;*
   b) *its transmission power is one of the following three discrete power points*

$$\left\{ \begin{array}{c} 0, \min\left\{p^{MAX},\ \frac{I_m}{g_m} - p_0^m\right\}, \\ \left[\frac{\phi_m h_{(1)}^m - \nu a_m n_m'}{(\nu - \phi_m) a^m h_{(1)}^m}\right]_0^{\min\left\{p^{MAX},\ \frac{I_m}{g_m} - p_0^m\right\}} \end{array} \right\} \tag{12}$$

which maximizes U_m. $h_{(1)}^m \ge h_{(2)}^m \ge \cdots \ge h_{(N_m)}^m$, $a_m = \frac{\gamma_0^m}{p_0^m}$, and $[a]_b^c$ is the projection of a onto the interval $[b, c]$.

Proof: Due to limited space, we omitted the proofs in [11]. □

Based on the above theorem, we propose an optimal power allocation Algorithm 1 to maximize U_m.

Algorithm 1

1) Sort the GUs with h_i^m in the descending order
2) Initialize $U_m^\star$ and $R_m^\star$ using (12)
3) For $i = 2 : 1 : N_m$
4) If $(i-1)\,p^{\mathrm{MAX}} \le \frac{I_m}{g_m} - p_0^m$, update

$$p_j^m = \begin{cases} p^{\mathrm{MAX}} \text{ if } j = 1, 2, \cdots, i-1 \\ \min\left\{p^{\mathrm{MAX}}, \frac{I_m}{g_m} - p_0^m - (i-1)\,p^{\mathrm{MAX}}\right\} \text{ if } j = i \\ 0 \text{ if } j = i+1, i+2, \cdots, N_m \end{cases}$$

and calculate R_m (7) and U_m (8). If $U_m \ge U_m^\star$, $U_m^\star = U_m$ and $R_m^\star = R_m$.
5) Else, terminate.
6) End
7) Output: $U_m^\star$ and $R_m^\star$

B. Individualized Refunding at MNO

The problem $\mathcal{P}_{\mathrm{L}}$ is generally difficult to solve since $R_m(\mathbf{p}^m)$ is a non-convex and implicit function of ϕ_m and I_m. Therefore, we propose a look-up table approach to determine individualized refunding and interference temperature constraints to different SHs. The MNO divides the feasible refunding interval $[0, \pi]$ into T equal intervals with the step size $\Delta\pi = \frac{\pi}{T}$ and the aggregate interference interval $[I_0, Q]$ into L equal intervals with step size $\Delta I = \frac{Q - I_0}{L}$, where $I_0 = \max(g_m p_0^m)$. Then each SH calculates and feedbacks a table of Ω_m in terms of different pairs of $(\phi_m = t\Delta\pi, I_m = I_0 + l\Delta I)$, $t = 0, 1, 2, \cdots, T$ and $l = 0, 1, 2, \cdots, L$. Finally, the MNO decides its strategy using a look-up table approach. The performance of Algorithm 2 is dependent on step size and search method, which is a tradeoff between optimality and computational complexity. As $\Delta\pi \to 0$ and $\Delta I \to 0$, the solution converges to subgame perfect equilibrium.

IV. Simulation Results

In this section, the performance of the proposed algorithms is investigated. The coverage of each small cell is a circular area and the small cells are separated from each other at least by 5 m. The GUs are randomly located in the coverage of small cells. The AWGN noise power σ^2 is -40 dB, the aggregate interference constraint $Q = 5 \times 10^{-3}$ W. The maximum transmission power is $p^{\mathrm{MAX}} = 3$ W and the path loss coefficient α_i is set to 3.

Algorithm 2 Look-up Table Approach

1) The MNO distributes π, Q, I_0, T and L to each SH;
2) The m^{th} $(m \in \mathcal{M})$ SH calculates and feedbacks the best response function table Ω_m for different pair of $(t\Delta\pi, I_0 + l\Delta I)$ using Algorithm 1, $t = 0, 1, 2, \cdots, T$ and $l = 0, 1, 2, \cdots, L$;
3) At MNO, it searches for the near-optimal strategies by any fast search method.

A. Typical small cell holder

We consider the refunding and optimization for a typical small cell. The target SINR and fixed transmission power for HU is 0 dB and $p_0 = p^{\mathrm{MAX}}$. There are $N = 6$ GUs. The channel gains of HU and GUs are generated as a sequence of i.i.d. exponential random variables with unit mean.

In Fig. 2, we illustrate typical SH's utility and GUs' sum-rate in terms of refunding ϕ_m under different interference temperature constraints I_m. With fixed I_m, the SH's utility is concave in terms of ϕ_m. Note that the GUs' sum-rate becomes constant after a threshold $\phi_m = 2$, which is due to interference temperature constraint. With fixed ϕ_m, the SH's utility becomes higher with a larger I_m. However, after a threshold $I_m = 6 \times 10^{-3}$ W, the SH's utility cannot be further increased. The total transmission powers $\sum_{i=1}^{N_m} p_i^m$ are bounded by I_m as well as p^{MAX} (11), hence, when $I_m \le 6 \times 10^{-3}$ W, increasing I_m corresponds to increasing $\sum_{i=1}^{N_m} p_i^m$, resulting in a higher sum-rate. When $I_m \ge 6 \times 10^{-3}$ W, the sum-rate is not affected by I_m but by p^{MAX}.

B. Single MNO with two small cells

We consider one MNO with two SHs. The distance between two small cells is 25 m and hence $g = \frac{1}{25^3}$. The usage-based fee is $\pi = 5$ and step sizes are $\Delta\pi = 0.5$ and $\Delta I = 3.2 \times 10^{-5}$ W. The reason why we choose above step sizes are that a finer step size cannot improve results and a coarser step size will distort the result. We assume the first SH has a larger remaining backhaul capacity ($\bar{C}_1 = 10$ bits/Hz/s) while the second SHs has a smaller remaining backhaul capacity ($\bar{C}_2 = 1$ bits/Hz/s).

In Fig. 3, we illustrate the MNO's net revenue in terms of refunding under different interference temperature constraints. The optimal strategy for the MNO is to set $\phi_1 = 2$ and $I_1 = 3 \times 10^{-3}$ W to the first SH and $\phi_2 = 1$ and $I_2 = 1.5 \times 10^{-3}$ W to the second SH. The more

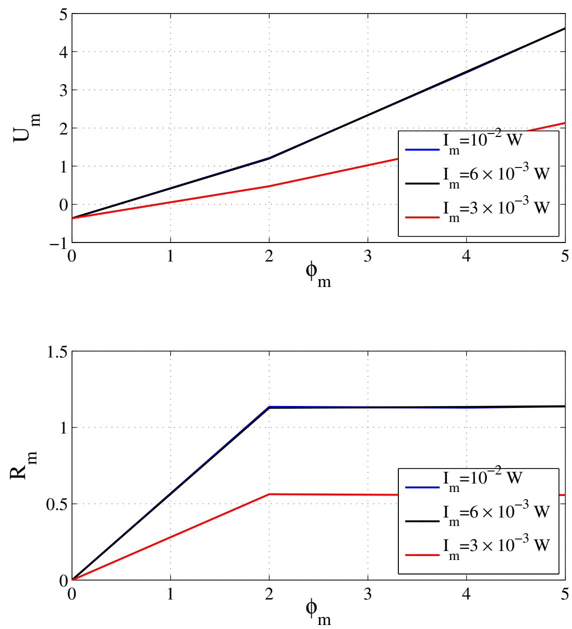

Figure 2. Typical SH's utility and GUs' sum-rate with refunding.

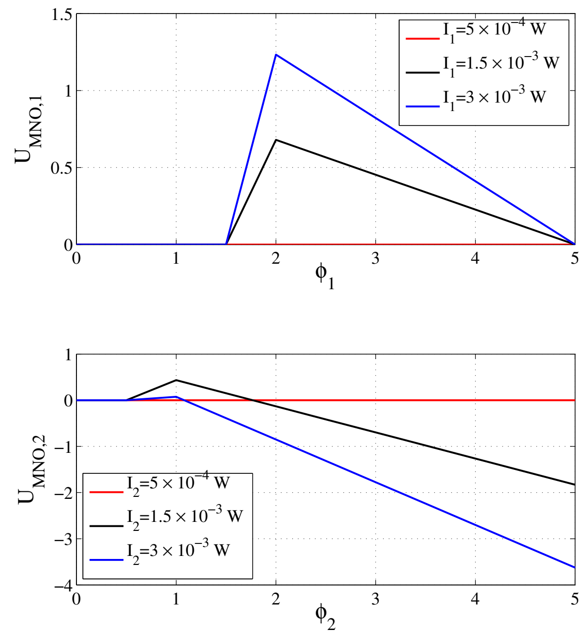

Figure 3. MNO's net revenue with refunding.

remaining backhaul capacity one SH obtains, the more it contributes to the MNO's net revenue. In return, the MNO refunds more to it. With fixed ϕ_m, the MNO can also adjust interference temperature constraint to further maximize its net revenue. At the first SH with $\phi_1 = 2$, the MNO can increase I_1 to achieve a higher net revenue. At the second SH with $\phi_2 = 1$, the MNO actually have to decrease I_2 to achieve a higher net revenue.

V. Conclusion

In this paper, we consider the optimization and refunding for small cells with limited-capacity backhaul. We proposed a refunding framework which significantly improves the MNO's net revenue and the SHs' utilities. It is a win-win strategy to provide refunding by MNO and SH's sharing remaining backhaul capacity with GUs. In the long-run, the MNO can determine the individualized refunding merely based on the small cell's remaining backhaul capacity after averaging enough channel realizations. In summary, this works provides an initial look into the importance of motivating small cell networks while considering the effect of limited-capacity backhaul.

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