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MONITORING FAULT CONDITION DURING MANUFACTURING USING THE KARHUNEN-LOÈVE TRANSFORM

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ABSTRACT

Monitoring the condition of parts and machine components is a crucial task in ensuring fault-free manufacturing. In this work, we propose an alternative condition monitoring technique, with great potential in extracting and isolating individual fault patterns from manufacturing signals. We propose that the Karhunen-Loève transform provides the ability to decompose measured signals into decorrelated fault patterns, in the form of fundamental eigenvectors. These fundamental eigenvectors can then be monitored by means of coefficient vectors, which indicate any changes in the fault patterns. The technique can provide accurate fault information, whether the manufacturing signals are deterministic, stochastic, stationary, or nonstationary.

This paper presents the fundamentals of the proposed technique and its extension to condition monitoring. The outputs of the Karhunen-Loève transform are studied to interpret their physical significance. Then, a subset of general manufacturing signals is used to understand the mathematical foundations of the technique. Extensions to general functions are investigated by means of numerical simulations. The technique proposed in this paper has great potential in providing a robust condition monitoring tool.

INTRODUCTION

Machine condition monitoring is one of the most crucial tasks in part manufacturing (Zhang et al., 1995). As we move towards intelligent, automated manufacturing, assuring the fault-free manufacturing of a part becomes a challenging task. Fault-free part production is achieved by continually monitoring the condition of the manufacturing process and the manufacturing machine. The purpose is to detect potential faults in the process or machine before damage becomes costly. Diagnosis of the fault origins typically follows fault detection, bringing important savings in time and money (Eppinger et al., 1995; Sottile and Holloway, 1994).

This paper focuses on monitoring dynamic signals from manufacturing processes. Examples of dynamic signals are vibration from machine components and surface profiles from manufactured parts. Monitoring manufacturing signals provides great insight on part and machine condition. Specifically, we currently focus on monitoring the condition of the surface of a part in order to achieve two goals: to assure that the part will not get scraped or refabricated; and to assure that the manufacturing machine does not fail during operation. We accomplish the first goal by collecting sample surface profile measurements on a regular basis during part production. Any faults or deviations in the dynamics of the machine or its submechanisms are assumed to leave a "fingerprint" on the surface being manufactured (Whitehouse, 1994). The collected surface profiles are then analyzed using signal processing tools to extract information about the fault condition of the part. We accomplish the second goal by studying the characteristics of individual fault patterns. The shape of each individual pattern can then be used to diagnose the cause of the faults.

The field of fault detection and diagnosis constantly searches for better tools to accomplish the above tasks accurately. The most commonly used tools often result in obscured information about the fault condition and result in frequent false alarms, delayed warnings about the fault condition, or insensitivity to a legitimate fault condition (Spiewak, 1991; Whitehouse, 1994). Accurate analysis of dynamic signals requires the use of signal processing tools. However, poor performance is often related to the careless selection of signal processing algorithms used to extract the fault features. Specifically, it becomes difficult to detect the proper fault features in the presence of obscuring effects such as averaging the time-varying components (nonstationarities).

The ultimate goal of this work aim to assure the automated production of fault-free parts. We claim that a robust fault detection and diagnosis method is necessary to satisfy this goal. The robust method must be able to analyze any type of signal and extract independent fault features in an accurate fashion. In addition, the method must be a unified tool that can be used in any fault situation. Finally, the method must enable the diagnosis of a machine without prior knowledge of the possible fault patterns. Specifically, we have to enable the study of the "shape" of each fundamental fault pattern individually in order to understand their characteristics, quantify their severity, and diagnose their origin. A good example is when a new system is being studied for possible faults. In such a case, there is no prior experience about the possible faults, or fault patterns. Such a scenario presents us with a very challenging problem as it requires the detection and diagnosis of any type of fault, without prior knowledge of the expected faults.

Signals in Manufacturing

Signals can be categorized as deterministic or stochastic. A good example of deterministic signals is the case of periodic signals (Bendat and Piersol, 1986; Braun, 1986). These signals may typically be detected, since they follow a known model by which exact future values can be predicted. Stochastic signals, on the other hand, do not typically follow a known model, and their structure and future values can only be described by probabilistic statements (Braun, 1986). Stochastic signals can exhibit stationary and/or nonstationary characteristics. The probabilistic laws describing stationary random signals are time-invariant, i.e., the statistical properties of the signal do not change with time (Braun, 1986). However, signals from manufacturing machines may contain time-varying or nonstationary characteristics, where the statistical parameters change significantly with time, and thus can no longer be predicted by the common techniques (Braun, 1986; Whitehouse and Zheng, 1992). Nonstationary profiles can typically be regarded as deterministic factors operating on otherwise stochastic random processes (Bendat and Piersol, 1986; Box et al., 1994). Nonstationarities can be categorized in the form of three major types (Bendat and Piersol, 1986; Box et al., 1994): (1) patterns with a time-varying mean value; (2) patterns with a time-varying mean-square value (i.e., variance); and, (3) patterns with a time-varying frequency structure.

Most signals from real systems contain a combination of deterministic and stochastic characteristics, as well as stationary and nonstationary characteristics. Stochastic (random) patterns might be due to noise in signals, as well as due to the mechanism generating the signal, such as the abrasive wheel in a grinding process. Deterministic patterns might be obscured due to the random and nonstationary nature of signals, making their detection more difficult. Similarly, nonstationary features, such as freak marks on the surface of a manufactured part due to the inhomogeneity of the material, can also be obscured by other types of signals, including the random sources.

Signal Decomposition

One of the most crucial tasks in fault detection is the accurate decomposition of a complex signal into its decorrelated fault patterns. To obtain a decorrelated decomposition of the measured signal, we need to apply a mathematical transformation to the signal (Ahmed and Rao, 1975; Akansu and Haddad, 1992; Whitehouse, 1994). The aim of signal analysis is to extract relevant information from a signal by transforming it (Rioul and Vetterli, 1991). The Fourier transform is such a tool, resulting in a decomposition into different frequency components based on sines and cosines as basis vectors (Berry, 1991; Jones, 1994; Whitehouse, 1994). However, the decomposition is not accurate in the presence of nonstationary features, including linear trends and offset changes. In addition, accurate analysis of the exact shape of individual fault patterns is not feasible due to a loss of information via the transform. Specifically, when dealing with real data, nonstationarities and linear trends often appear falsely as additional frequency components, and modify the magnitudes of the remaining frequency components.

Attractive alternatives to the Fourier transform are the Wigner-Ville transform, the wavelet transform, and the higher-order spectral transform. The Wigner-Ville transform has been previously used in analyzing manufacturing signals, including surface signals (Boashash, 1992; Rohrbaugh, 1993; Whitehouse and Zheng, 1992). However, this transformation typically introduces redundant features that obscure the significant fault features. The wavelet transform has been used in a number of signal processing applications, including a few manufacturing applications, but there are no applications to surface profiles (Geng and Qu, 1994; Ladd, 1995). Wavelet transforms eliminate the resolution and accuracy problems of other time-frequency decompositions, while providing a more flexible time-scale resolution (Geng and Qu, 1994; Ladd, 1995). However, it is difficult to attribute physical meaning to the features (wavelet coefficients) extracted from a wavelet decomposition. In our work, we want to allow physical interpretation of the extracted features to enable understanding of the shape of the fault patterns, their status, and severity. Higher-order spectral transforms have been used in numerous signal processing applications, including several efforts in condition monitoring in manufacturing, with no applications to surface profiles (Barker et al., 1993; Fackrell et al., 1994). This method is used to detect unusual frequency components that would not have been detected with the second-order Fourier spectrum, maintaining phase information in addition to the magnitude information provided by the second-order spectral methods. However, it is often difficult to isolate and interpret the features corresponding to nonstationary signals. In addition to these transforms, there exist other transforms, including Gabor transforms, Walsh transforms, and Haar transforms (Kozek, 1993; Whitehouse, 1994), but these are simply extensions of the Fourier, Wigner-Ville, and wavelet transforms, so their utility is similar.

In this work, we investigate an alternative candidate, namely, an orthogonal transform called the Karhunen-Loève (KL) transform. This transform decomposes the signals into completely decorrelated components in the form of empirical basis functions that contain the majority of the variations in the original data. The KL transform has been used in many statistical applications; the more recent applications include recognition of faces in vision and turbulent structures in fluids. However, this powerful tool has not been used in condition monitoring applications. The KL decomposition can be applied to any type of signal, without prior knowledge of the characteristics of the fault patterns, which is a superior property of the Karhunen-Loève transform. Typically, no physical significance is attributed to the resulting features. We show that the KL decomposition can be used effectively to understand the fault mechanisms in manufacturing. In this light, we study the potential of the KL transform in this context, with the purpose of advancing the research in this field. Our ultimate goal is to develop a powerful fault condition monitoring technique.

KARHUNEN-LOÈVE TRANSFORM FOR CONDITION MONITORING IN MANUFACTURING

Although not found in the fault detection and diagnosis literature, the Karhunen-Loève transform (KL), is used in a variety of signal processing applications (Akansu and Haddad, 1992; Algazi et al., 1993; Ball et al., 1991; Fukunaga, 1972; Graham et al., 1993; Sirovich and Keefe, 1987; Therrien, 1992; Zahorian and Rothenberg, 1981). The transform has been used widely in the literature as a means of detecting "dominant" characteristics in a set of signals. Here, we present a small subset of the example applications of the transform.

In the field of speech signal analysis and synthesis, Zahorian uses Principal Components Analysis (PCA), an ancestor of the Karhunen-Loève transform, to select the most dominant characteristics from the spectral analysis of speech signals (Zahorian and Rothenberg, 1981). This study exploits the experimentally observed correlations among spectral band energies at different frequencies in order to derive a much smaller set of statistically independent parameters (principal components) which retain most of the information present in the original speech spectra (Zahorian and Rothenberg, 1981). Sirovich et al. apply the KL transform to pattern recognition, attacking the general problem of characterizing, identifying, and distinguishing individual patterns drawn from a well-defined class of patterns (Sirovich and Keefe, 1987). The patterns are pictures of faces of people, taken from a random sample of males. They show that a compressed version of the original faces can be reconstructed with reduced dimensionality to generate recognizable faces (Sirovich and Keefe, 1987). Ball et al. use the KL transform in the analysis of low-Reynolds number turbulent channel flow. Snapshots of the velocity field $u(x,t)$ are taken and the entire ensemble of velocity measurements are decomposed into spatial modes (basis eigenvectors) and time-dependent amplitude coefficients (Ball et al., 1991). Graham et al. use principal orthogonal decomposition to decompose spatiotemporal signals into orthogonal spatial components and time-dependent amplitudes. Patterns are obtained during the oxidation of hydrogen on a nickel disk in a mixed flow reactor (Graham et al., 1993). Finally, Algazi et al. use the KL transform to capture the time-varying structure of the spectral envelope of speech. Acoustic subword decomposition using fast Fourier transform (FFT) and the KL transform are used to extract and efficiently represent the highly correlated structure of the spectral envelope (Algazi et al., 1993).

The Karhunen-Loève transform of manufacturing signals has never been proposed as an alternative fault condition monitoring technique, partially because of the difficulty in understanding the physical significance of the resulting outputs in the transform domain. In the following, we first present the mathematical basis of the Karhunen-Loève transform. Then we present our extensions to the transform in an attempt to attribute physical significance to its outputs. These extensions constitute the foundation for developing an effective condition monitoring tool.

Mathematical Foundation

In this subsection, we summarize the background of the Karhunen-Loève approach. A random process $x(t)$, defined in the time domain (or spatial domain) $(0, T)$, can be expressed as a linear combination of orthogonal functions (Fukunaga, 1972):

$$\mathbf{x}(t) = \sum_{i=1}^{N} \mathbf{y}_i \varphi_i(t), \quad (1)$$

where the orthogonal functions $\varphi_i(t)$ are deterministic functions, and the ith coefficient $\mathbf{y}_i$ is a random variable. Equation 1 is the Karhunen-Loève expansion of the original data, composed of a linear combination of the basis functions and the feature coefficients in the transformed domain.

In the discrete time domain, we take n sampled values of the time functions and convert them to vectors as shown in the following equations (Fukunaga, 1972):

$$\mathbf{X} = [\mathbf{x}(t_1) \ldots \mathbf{x}(t_n)]^T \quad (2)$$

$$\mathbf{\Phi} = [\varphi_i(t_1) \ldots \varphi_i(t_n)]^T, \quad (3)$$

where each time-sampled value $\mathbf{x}(t_i)$ is a random variable.

Assuming zero-mean random processes, the covariance matrix equals the autocorrelation matrix, and is computed as $R(t, \tau) = E[\mathbf{x}(t)\mathbf{x}^*(\tau)]$. If $\varphi_i(t_k)$ are the eigenfunctions of $R(t_l, t_k)$, they must satisy the following characteristic equation:

$$\sum_{k=1}^{n} R(t_l, t_k)\varphi_i(t_k) = \lambda_i \varphi_i(t_l), \quad (4)$$

where $i, l = 1, 2, \ldots, n$. Equation 4 can be written in matrix form to define the eigenvalues λ_i and eigenvectors $\mathbf{\Phi}_i$ as follows:

$$\mathbf{S}\mathbf{\Phi}_i = \lambda_i \mathbf{\Phi}_i, \quad (5)$$

where $i = 1, 2, \ldots, n$ and $\mathbf{S}$ is the $n \times n$ covariance matrix defined by:

$$\mathbf{S} = \begin{bmatrix} R(t_1, t_1) \cdots R(t_1, t_n) \\ \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \\ R(t_n, t_1) \cdots R(t_n, t_n) \end{bmatrix} \quad (6)$$

The covariance matrix equation is solved to obtain its eigenvalues and corresponding eigenvectors. The eigenvectors are orthonormal and hence need to satisfy $\mathbf{\Phi}^T \mathbf{\Phi} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The eigenvalues are ranked in order of significance, as compared to the total energy represented by the sum of all eigenvalues. The eigenvectors corresponding to the dominant eigenvalues represent the fundamental functions in the transformed domain. The KL coefficients y_i are computed by projecting the original data onto the new domain, represented by the dominant basis functions:

$$\mathbf{y}_i = \mathbf{\Phi}_i^T \mathbf{X}, \quad (7)$$

where we have one coefficient vector corresponding to each eigenvector. The resulting vector $\mathbf{Y} = [\mathbf{y}_i \ldots \mathbf{y}_n]^T$ contains the n coefficient vectors corresponding to the n eigenvectors $\mathbf{\Phi} = [\varphi_1 \ldots \varphi_n]^T$. $\mathbf{Y}$ is an orthonormal transformation of the random vector $\mathbf{X}$ into the new vector space, and is itself a random vector. We can reconstruct the original data in the new transformed function domain as a linear combination of the fundamental eigenvectors and corresponding coefficient vectors as in $\mathbf{X} = \mathbf{\Phi}\mathbf{Y} = \sum_{i=1}^{m} \mathbf{y}_i \varphi_i$, where $m < n$ is the number of fundamental eigenvectors.

Extension to Condition Monitoring

In order to enable the use of the KL transform for effective condition monitoring, we claim that the KL transform results in individual patterns, regardless of the type of signal. These decomposed patterns correspond to the fundamental modes in the manufacturing system and can be monitored individually for the occurrence of potential faults. The second claim we make in this work is that, given the individual fundamental patterns of the measured signal, we can use the corresponding coefficient vectors to monitor the change in time of each pattern. This result will enable the monitoring of fault occurrences in a manufacturing process.

The first step towards enabling condition monitoring via the KL transform is understanding what each of the KL outputs signify. The first step towards understanding the outputs is to predict the fundamental patterns and their coefficient vectors given a known set of signals. In this light, we first present an interpretation of the outputs from the KL transform and the manner in which they will be used for effective condition monitoring.

The next step relies on understanding the mechanics of the KL transform. We present a simple case using linear vectors to prove our claims. To verify the first claim, we first have to show that, given a set of input vectors with known patterns, the transform results in accurate fundamental patterns. In other words, given a single-component signal consisting of a linear vector, the resulting fundamental pattern must be a linear vector, or, given a pure sinusoidal, the resulting fundamental pattern must be a sinusoidal function, etc. In this paper, the case of linear vectors is presented in detail. Once we understand the mechanics of the KL transform using single-component inputs, we then need to show that given multi-component input signals (as opposed to pure signals with a single pattern only), the resulting fundamental patterns will correspond to each of the significant fault patterns in the signal. Discussion of the results and the necessary extensions to general functions are presented last.

Covariance Matrix, Eigenvectors, and Coefficient Vectors

Given input data $\mathbf{X} = [\mathbf{x}(t_1)\ldots\mathbf{x}(t_N)]^T$, the covariance matrix of $\mathbf{X}$ is $\mathbf{S}_X = E[(\mathbf{X}-\bar{\mathbf{X}})(\mathbf{X}-\bar{\mathbf{X}})^T]$, where $E[\cdot]$ is the expected value operator, and $\bar{\mathbf{X}}$ is the mean vector $E[\mathbf{X}]$. We remove the mean vector from the input vectors, transforming the input data into zero-mean, i.e., $\bar{\mathbf{X}} = 0$. We have $j = 1, \ldots, M$ input data vectors $\mathbf{X}_j$, each of $i = 1, \ldots, N$ sampled random variables. The M input vectors represent the "snapshots" of the process being monitored, taken at regular time intervals. The input matrix $\mathbf{X}$ is composed of these M snapshots $\mathbf{X}_j$. Estimating the expected value operator as an ensemble average, the covariance matrix is then computed as:

$$\hat{\mathbf{S}} = \frac{1}{M}\sum_{j=1}^{M}\mathbf{X}_j\mathbf{X}_j^T \tag{8}$$

$$= \frac{1}{M}\left(\begin{bmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1N}\end{bmatrix}\begin{bmatrix} x_{11}x_{12}\cdots x_{1N}\end{bmatrix} + \cdots + \right.$$

$$\left. + \cdots + \begin{bmatrix} x_{M1} \\ x_{M2} \\ \vdots \\ x_{MN}\end{bmatrix}\begin{bmatrix} x_{M1}x_{M2}\cdots x_{MN}\end{bmatrix}\right)$$

$$= \frac{1}{M}\left(\begin{bmatrix} x_{11}^2 & x_{11}x_{12} & \cdots & x_{11}x_{1N} \\ x_{12}x_{11} & x_{12}^2 & \cdots & x_{12}x_{1N} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ x_{1N}x_{11} & x_{1N}x_{12} & \cdots & x_{1N}^2 \end{bmatrix} + \cdots + \right.$$

$$\left. + \cdots + \begin{bmatrix} x_{M1}^2 & x_{M1}x_{M2} & \cdots & x_{M1}x_{MN} \\ x_{M2}x_{M1} & x_{M2}^2 & \cdots & x_{M2}x_{MN} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ x_{MN}x_{M1} & x_{MN}x_{M2} & \cdots & x_{MN}^2 \end{bmatrix}\right)$$

$$= \frac{1}{M}\left(\begin{bmatrix} s_{11} & s_{12} & \cdots & \cdots \\ \cdots & s_{22} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & s_{MN} \end{bmatrix}\right)$$

where

$$s_{11} = x_{11}^2 + x_{21}^2 + \cdots + x_{M1}^2 \tag{9}$$

$$s_{12} = x_{11}x_{12} + x_{21}x_{22} + \cdots + x_{M1}x_{M2} \tag{10}$$

$$\vdots \tag{11}$$

$$s_{22} = x_{12}^2 + x_{22}^2 + \cdots + x_{M2}^2$$

$$\vdots \tag{12}$$

$$s_{MN} = x_{1n}^2 + x_{2N}^2 + \cdots + x_{MN}^2$$

The diagonal terms represent the average variances of the individual random variables, while the off-diagonal terms represent the average covariances between two random variables. The first term of the first row in the matrix is the variance of the first point for each input vector, the second term is the covariance between first and second sampled points for each vector, the third term is the covariance between the first and third sampled points for each input vector, etc. Note that if the input vectors are not zero-mean, these terms are interpreted as the mean-squared deviations from the mean vector.

The resulting N by N covariance matrix is real and symmetric, and contains the mean-squared deviations from the mean vector. If the input vectors are zero-mean, then the matrix contains mean-squared deviations between each input vector. As a result, the matrix contains all of the variational structure of the input data. The eigenvectors of this matrix provide the principal axes of the variational structure, presented in the form of basis functions representing the dominant patterns in the data. The eigenvectors are used here to determine the dominant patterns in the monitored signal. The eigenvalues of the covariance matrix are computed using $det(\mathbf{S}-\lambda\mathbf{I}) = 0$, where $\mathbf{I}$ is the identity matrix, and λ are the roots of the characteristic polynomial, or the eigenvalues. The eigenvectors $\mathbf{\Phi}_i$ of the matrix are then computed using: $(\mathbf{S}-\lambda\mathbf{I})\Phi_i = 0$. We will show that these eigenvectors are dependent on the deviations of each point of the input vectors from the mean vector.

Even though the covariance matrix has dimension N, the rank of the matrix is determined by the number of input vectors M, resulting in M independent eigenvectors of dimension N. Each eigenvector $\mathbf{\Phi}_i$ has a coefficient vector $\mathbf{Y}_i$ associated with it, computed using $\mathbf{Y}_i = \mathbf{\Phi}_i^T\mathbf{X}$. The coefficient vectors, of dimension M, are the projection of the original data matrix $\mathbf{X}$ onto the corresponding eigenvectors, and hence represent the weight of each input vector in the new transform domain spanned by the eigenvectors $\mathbf{\Phi}_i$. As a result, the coefficient vectors are used here to monitor the changes over time for each dominant eigenvector. The eigenvalues λ_i corresponding to each eigenvector $\mathbf{\Phi}_i$ represent the variance of the corresponding coefficient vectors $\mathbf{Y}_i$, i.e., $E[\mathbf{Y}_i^2] = \lambda_i$. Hence, the value of the eigenvalues will indicate the significance of each fundamental pattern in the new transform domain, as well as reflect any changes in the fundamental patterns.

Representing Manufacturing Signals as General Functions

In this work, we view a manufacturing signal as being potentially composed of any type of signal. A general function $g(x,t)$ represents a manufacturing signal. This function is composed of a multitude of functions $f_1(x,t)$, $f_2(x,t)$, etc. In reality, the exact shape of the functions f_i, and the exact nature of the interactions between these functions are not known *a priori*. The decomposition of $g(x,t)$ into individual functions f_i will ultimately enable accurate monitoring. In this paper, we assume that $g(x,t)$ is composed of a linear combination of known signal types. We use this knowledge to study the detection and extraction properties of the Karhunen-Loève transform as an alternative condition monitoring technique.

Example: Monitoring Linear Trends

Linear vectors are the simplest form of deterministic functions. As a result, the decomposition of general functions is first studied in terms of linear vectors. The study of this simple case allows a clear demonstration of the mechanics of the KL transform in condition monitoring. The extension to general function decomposition is discussed in a subsequent section.

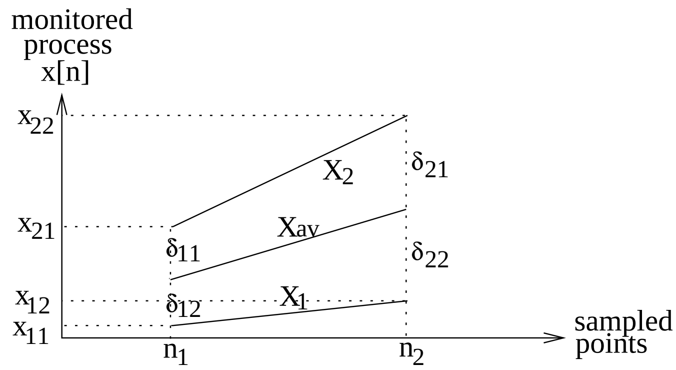


Figure 1. Simple Case: Linear Vectors

Assumptions: Two sample input vectors $\mathbf{X_1}$ and $\mathbf{X_2}$, each with two sample points $(x_{11}\,x_{12})^T$ and $(x_{21}\,x_{22})^T$, respectively, are collected, as shown in Figure 1 (M=2, N=2). The input vectors represent pure linear trends (straight lines), of increasing slopes. The mean vector $\mathbf{X}_a = (x_{a1}\,x_{a2})^T$ is also a straight line, with 2 sampled points. δ_{ij} is assumed to be the deviation of the ith sampled point of the jth input vector from the mean vector. In this case, since we only have two input vectors, the average vector is equidistant from each input vector, i.e., $\delta_{11} = \delta_{12}$ and $\delta_{21} = \delta_{22}$, as shown in Figure 1.

Conjecture 1 Let $\mathbf{X_1} = [x_{11}\,x_{12}]^T$ and $\mathbf{X_2} = [x_{21}\,x_{22}]^T$ be 2 linear vectors with 2 points each, representing 2 straight lines with different slopes. The KL transform results in a single fundamental eigenvector $\mathbf{\Phi} = [\varphi_1\,\varphi_2]^T$, which is a straight line.

Proof Let $\mathbf{X_1} = [x_{11}\,x_{12}]^T$ and $\mathbf{X_2} = [x_{21}\,x_{22}]^T$ be two linear vectors. The mean vector is $\mathbf{X}_a = (x_{a1}\,x_{a2})^T = [\frac{x_{11}+x_{21}}{2}\;\frac{x_{12}+x_{22}}{2}]^T$. Subtracting the mean vector from the input vectors, we obtain the zero-mean input vectors, used to compute the covariance matrix:

$\mathbf{Z_1} = \mathbf{X_1} - \mathbf{X}_a = [x_{11} - x_{a1}\;\; x_{12} - x_{a2}]^T$, and,

$\mathbf{Z_2} = \mathbf{X_2} - \mathbf{X}_a = [x_{21} - x_{a1}\;\; x_{22} - x_{a2}]^T$.

Assuming δ_{ij} deviations of the input vectors from the mean vector, the zero-mean input vectors become:

$\mathbf{Z_1} = [-\delta_{11}\;\; -\delta_{21}]^T$, and,

$\mathbf{Z_2} = [\delta_{11}\;\delta_{21}]^T$.

The data matrix becomes $\mathbf{Z} = [\mathbf{Z}_1\,\mathbf{Z}_2]$.

The covariance matrix is computed using:

$\hat{\mathbf{S}} = \frac{1}{2}\sum_{i=1}^{2}\mathbf{Z}_i\mathbf{Z}_i^T$, which becomes:

$$\hat{\mathbf{S}} = \frac{1}{2}\left[\begin{bmatrix}-\delta_{11}\\-\delta_{21}\end{bmatrix}\begin{bmatrix}-\delta_{11} & -\delta_{21}\end{bmatrix} + \begin{bmatrix}\delta_{11}\\\delta_{21}\end{bmatrix}\begin{bmatrix}\delta_{11} & \delta_{21}\end{bmatrix}\right] \quad (13)$$

$$= \begin{bmatrix} \delta_{11}^2 & \delta_{11}\delta_{21} \\ \delta_{21}\delta_{11} & \delta_{21}^2 \end{bmatrix}$$

The eigenvalues are computed using $det(\mathbf{S} - \lambda \mathbf{I}) = 0$, which becomes:

$$\begin{vmatrix} \delta_{11}^2 - \lambda & \delta_{11}\delta_{21} \\ \delta_{21}\delta_{11} & \delta_{21}^2 - \lambda \end{vmatrix} = 0 \qquad (14)$$

$$\Leftrightarrow (\delta_{11}^2 - \lambda)(\delta_{21}^2 - \lambda) - \delta_{11}^2\delta_{21}^2 = 0$$
$$\Leftrightarrow \lambda^2 - (\delta_{11}^2 + \delta_{21}^2)\lambda = 0$$

The solution of this characteristic polynomial in λ is $\lambda = 0$ or $\lambda = \delta_{11}^2 + \delta_{21}^2$, resulting in only one dominant eigenvalue. The corresponding eigenvector is computed using $(\mathbf{S} - \lambda \mathbf{I})\mathbf{\Phi}_i = 0$, which becomes:

$$\begin{aligned} (\mathbf{S} - \lambda \mathbf{I})\mathbf{\Phi}_i &= 0 \qquad (15) \\ &\Leftrightarrow \begin{bmatrix} \delta_{11}^2 - (\delta_{11}^2 + \delta_{21}^2) & \delta_{11}\delta_{21} \\ \delta_{21}\delta_{11} & \delta_{21}^2 - (\delta_{11}^2 + \delta_{21}^2) \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \\ &= \begin{bmatrix} -\delta_{21}^2 & \delta_{11}\delta_{21} \\ \delta_{21}\delta_{11} & -\delta_{11}^2 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = -\delta_{21}^2\varphi_1 + \delta_{11}\delta_{21}\varphi_2 \\ &= 0 \\ &\Leftrightarrow \varphi_1 = \frac{\delta_{11}}{\delta_{21}}\varphi_2 \end{aligned}$$

which is the equation for a straight line. Since the eigenvector has to be orthonormal, the orthonormality condition $\mathbf{\Phi}^T\mathbf{\Phi} = \mathbf{I}$ implies $\varphi_1^2 + \varphi_2^2 = 1$, resulting in $\varphi_2 = \frac{\delta_{21}}{\sqrt{\delta_{11}^2+\delta_{21}^2}}$ and $\varphi_1 = \frac{\delta_{11}}{\sqrt{\delta_{11}^2+\delta_{21}^2}}$, which is a normalized linear vector. []

Proposition 1.1 The resulting eigenvector is dependent only on the deviations of each input vector from the mean vector.

Proof The eigenvector from Conjecture 1 above is $\mathbf{\Phi} = [\varphi_1 \, \varphi_2]^T = [\frac{\delta_{11}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \; \frac{\delta_{21}}{\sqrt{\delta_{11}^2+\delta_{21}^2}}]^T$, which is a straight line whose coordinate values are only dependent on the deviations δ_{11} and δ_{21} of the input vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ respectively, from the mean vector $\mathbf{X}_a$. []

Proposition 1.2 If the input vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ are parallel to each other, the resulting fundamental eigenvector is a straight line of slope zero.

Proof If the input vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ are parallel to each other, $\delta_{11} = \delta_{21}$, and hence the components of the eigenvector become $\varphi_1 = \frac{\delta_{11}}{\delta_{21}}\varphi_2 = \frac{\delta_{11}}{\delta_{11}}\varphi_2 = 1\varphi_2$, which is a straight line of slope zero. []

Proposition 1.3 The coefficient vector $\mathbf{Y} = [y_1 \, y_2]^T$ corresponding to the fundamental eigenvector $\mathbf{\Phi} = [\varphi_1 \, \varphi_2]^T$ indicates the change in the slope between the two input vectors $\mathbf{X_1}$ and $\mathbf{X_2}$.

Proof The coefficient vector $\mathbf{Y} = [y_1 \, y_2]^T$ corresponding to the eigenvector $\mathbf{\Phi} = [\varphi_1 \, \varphi_2]^T = [\frac{\delta_{11}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \; \frac{\delta_{21}}{\sqrt{\delta_{11}^2+\delta_{21}^2}}]^T$ is computed using $\mathbf{Y} = \mathbf{\Phi}^T\mathbf{Z}$, which becomes:

$$\begin{aligned} [\, y_1 \;\; y_2 \,] &= \left[\frac{\delta_{11}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \;\; \frac{\delta_{21}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \right] \begin{bmatrix} -\delta_{11} & \delta_{11} \\ -\delta_{21} & \delta_{21} \end{bmatrix} \qquad (16) \\ &= \left[-\sqrt{\delta_{11}^2 + \delta_{21}^2} \;\; \sqrt{\delta_{11}^2 + \delta_{21}^2} \right] \end{aligned}$$

The components y_1 and y_2 can be plotted with respect to the number of input vectors to indicate a positive change between the two input vectors. The change between the two input vectors is equal to the difference $\mathbf{Z_2} - \mathbf{Z_1}$, which is equal to the sum of the δ_{ij} deviations from the mean vector:

$$\mathbf{Z_2} - \mathbf{Z_1} = \begin{bmatrix} \delta_{11} \\ \delta_{21} \end{bmatrix} - \begin{bmatrix} -\delta_{11} \\ -\delta_{21} \end{bmatrix} = \begin{bmatrix} 2\delta_{11} \\ 2\delta_{21} \end{bmatrix} \qquad (17)$$

Since $\mathbf{X_i} = \mathbf{Z_i} + \mathbf{X_a}$, we can see that $\mathbf{X_2} - \mathbf{X_1} = \mathbf{Z_2} - \mathbf{Z_1}$, which indicates the change between the two input vectors. []

Proposition 1.4 The original data vectors $\mathbf{X_1}$ and $\mathbf{X_2}$ can be reconstructed as a linear combination of the eigenvector $\mathbf{\Phi} = [\varphi_1 \, \varphi_2]^T$ and the coefficient vector $\mathbf{Y} = [y_1 \, y_2]^T$ in the transform domain.

Proof The zero-mean data matrices $\mathbf{Z_1}$ and $\mathbf{Z_2}$ are reconstructed using $\mathbf{Z}_1 = y_1\mathbf{\Phi}$ and $\mathbf{Z}_2 = y_2\mathbf{\Phi}$, which become:

$$\mathbf{Z}_1 = -(\sqrt{\delta_{11}^2 + \delta_{21}^2}) \begin{bmatrix} \frac{\delta_{11}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \\ \frac{\delta_{21}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \end{bmatrix} = \begin{bmatrix} -\delta_{11} \\ -\delta_{21} \end{bmatrix} \qquad (18)$$

$$\mathbf{Z}_2 = (\sqrt{\delta_{11}^2 + \delta_{21}^2}) \begin{bmatrix} \frac{\delta_{11}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \\ \frac{\delta_{21}}{\sqrt{\delta_{11}^2+\delta_{21}^2}} \end{bmatrix} = \begin{bmatrix} \delta_{11} \\ \delta_{21} \end{bmatrix} \qquad (19)$$

When the mean vector $\mathbf{X}_a$ is added to the zero-mean vectors $\mathbf{Z_1}$ and $\mathbf{Z_2}$, we obtain the original data vectors $\mathbf{X_1}$ and $\mathbf{X_2}$. []

Discussion: Detecting and Monitoring Linear Vectors

The insights gained from studying linear vectors can help in understanding the significance of the Karhunen-Loève transform. As demonstrated above, the transform amounts to solving a familiar eigenvector equation to determine the fundamental eigenvectors and the coefficient vectors in the new transform domain. Conjecture 1 implies that we can predict the eigenvectors resulting from the KL transform, if we know that the input vectors are linear. This result assures that we can extract dominant patterns, such as linear trends, in the form of individual fundamental eigenvectors. The next step is to prove that the same fundamental pattern can be extracted in the presence of other deterministic, stochastic, or nonstationary patterns. The conjecture further implies that we can attribute physical significance to the extracted patterns, relating them to faults in the system. Once similar proofs are performed for general functions, the KL transform will provide a complete picture of all the existing patterns, including linear trends, offsets, sinusoids, and nonstationarities. The characteristics of each individual pattern will help isolate the origin of the fault mechanism. Proposition 1.1 implies that, for the case of linear vectors, the KL eigenvectors will be dependent on the deviations between input vectors only. In terms of condition monitoring, this result potentially implies that all deviations from the "normal" state of operation in manufacturing will be detected with this method, hence detecting changes, indicative of faults.

The extraction of fundamental patterns is important in characterizing the nature of the fault patterns and isolating their potential origins. The other significant result stems from Proposition 1.3, implying that changes in the fundamental patterns can be monitored by observing the coefficient vectors. Once again, similar proofs are necessary for the case of more complex signals. Changes in each fundamental pattern can be monitored using the individual coefficient vectors. This result is an important consideration in declaring a fault. The severity of each individual pattern can be determined by studying the linearly separable coefficients over the snapshots.

Finally, Proposition 1.4 provides us with a means of reconstructing the fault patterns using a linear combination of KL eigenvectors and coefficient vectors. This result can be utilized in a very useful way. The manufacturing signals measured can be reconstructed after removing various patterns (such as stochastic and nonstationary effects) and the coherent modes (such as deterministic patterns) can be visualized more accurately. One of the problems with the Fourier transform, for example, is that the amplitude and frequency pairs extracted may be unreliable in the presence of linear trends, high noise, or nonstationarities. The KL transform has the potential of providing a clear picture of each individual pattern, or a combination of patterns, facilitating the characterization of the origin of each fault. The reconstructed patterns can be used for various applications, such as modeling of faults, training of neural networks, prediction of fault patterns given specific sources of faults, etc. (Tumer et al., 1995; Tumer et al., 1997a).

General Functions and Condition Monitoring

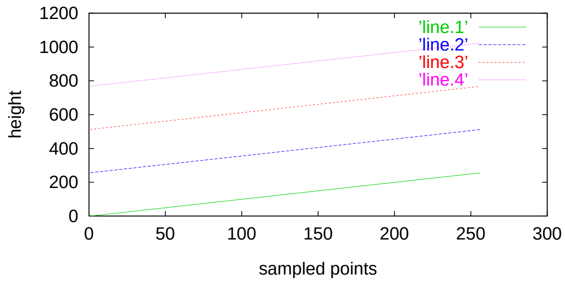
Detecting linear vectors constitutes the first step towards decomposing a general function $g(x,t)$ into its fundamental functions. Linear trends often obscure data; for example, linear trends appear as a low frequency component using Fourier transform techniques. Typically, preprocessing of the data is necessary to remove the linear trends by means of linear regression techniques. In this work, we aim to develop a unified decomposition method that will decompose the measured data into all of its fundamental components, including linear trends, sinusoidal patterns, stochastic patterns, and nonstationary patterns.

Before advancing to that step, however, it is important to numerically verify the results presented in the previous section. Similarly, before attempting to understand the mechanics of the KL decomposition using more complex signals, it is important to conduct a numerical study to determine the feasibility of the proposed method in decomposing such signals. In this light, the following section presents preliminary numerical results. More detailed simulation results are presented in a subsequent paper (Tumer et al., 1997b).

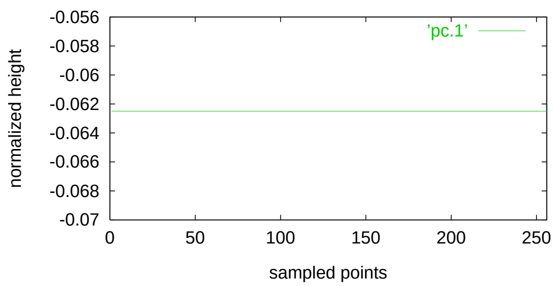
The ultimate goal of this work is to understand the mechanics of the KL decomposition so that a robust condition monitoring tool can be implemented. Note that, if we can predict the fundamental patterns from the decomposition of known general functions, we can attempt to undertake the more complex task of monitoring and detecting unexpected and unknown faults. We believe that the Karhunen-Loève transform can be used to effectively identify fundamental modes in the manufacturing system, and to monitor changes in each, indicating the occurrence of faults. The understanding of the basics described in this paper build the foundation for further investigating the Karhunen-Loève transform as a tool for condition monitoring.

Numerical Verification

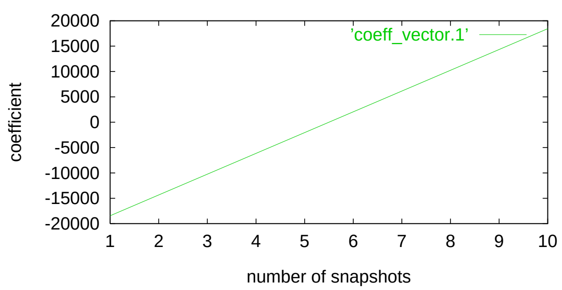
This section presents numerical simulation results using the Karhunen-Loève transform. First, the simple case of linear vectors is duplicated using input vectors with higher dimensionality ($M = 10$ inputs, with $N = 256$ sampled points each). Then, a more complex case of multi-component signals is simulated. The multicomponent signal is composed of 2 pure sinusoids of different frequency, a linear trend, and



(a) Input Profiles.



(b) Fundamental Eigenvector.



(c) Corresponding Coefficient Vector.

Figure 2. Simulation Results: Linear Vectors

random noise. The multicomponent signal is representative of a signal measured from a manufacturing machine. Accurate decomposition of such a signal will allow us to study each fault pattern separately and isolate their origin. More detailed simulation results are presented in (Tumer et al., 1997b).

The numerical simulations of the KL transform of linear vectors verify our analytical results. The linear vectors, shown in Figure 2a, result in the a single fundamental linear eigenvector, shown in Figure 2b, and the corresponding coefficient vector, shown in Figure 2c. As predicted analytically, the eigenvector is characteristic of the fundamental pattern, which is a straight line. Furthermore, the corresponding coefficient vector indicates the positive slope change among the input vectors (Figure 2a).

The multicomponent signal is composed of a combination of 2 sinusoidal patterns, a linear trend, and a stochastic mode. $M = 40$ snapshots are assumed to be collected from the manufacturing process, each with $N = 256$ sampled points, shown in Figure 3. The KL transform of the multicomponent signal results in 3 individual fundamental patterns, shown in Figures 4a and 4b, with the noise component filtered. The coefficient vectors corresponding to the 3 fundamental patterns show the changes in each fundamental pattern over the collected snapshots, as shown in Figures 4c and 4d. Figures 4a and 4b show the fundamental patterns corresponding to the linear trend, low-frequency sinusoid, and the high-frequency sinusoid. These individual patterns can then be monitored using the corresponding coefficient vectors. Figure 4c indicates the change in the slope of the linear trend. The coefficient vector in Figure 4d would indicate any change in the amplitude of the individual frequency components. Various other cases of multicomponent signal decomposition are presented in a subsequent paper (Tumer et al., 1997b), including examples of nonstationary changes in the monitored signals.

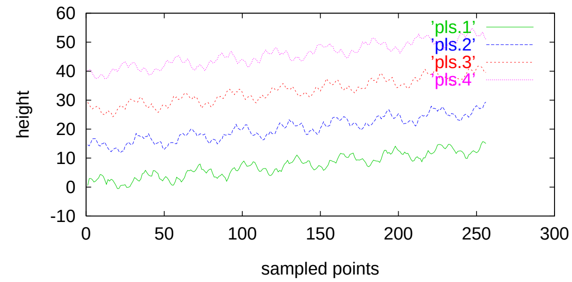
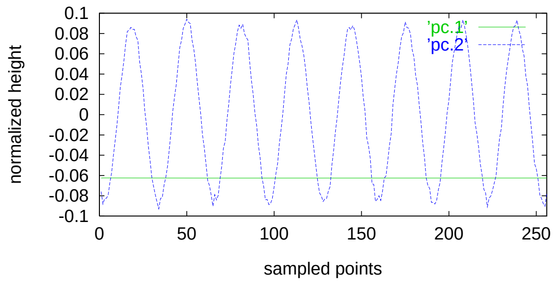


Figure 3. Input Profiles.

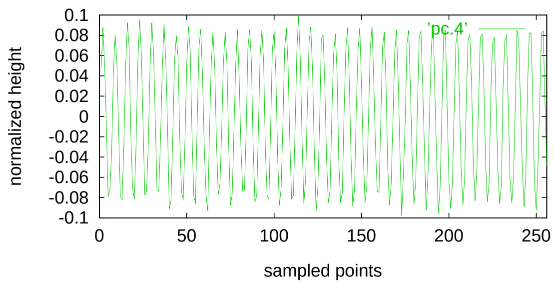
CONCLUSIONS

In this paper, an alternative machine condition monitoring method is proposed. The mathematical basics of the Karhunen-Loève transform for condition monitoring are developed. The method is shown to have great potential for condition monitoring. Specifically, decomposition of monitored manufacturing signals into its fundamental patterns is presented. These individual fundamental patterns can be monitored by means of the corresponding coefficient vectors, which indicate the change in each pattern over time. These changes are indicative of faults in the system. The physical significance of the KL results are explored, and the significance of fundamental fault patterns and the means of monitoring them are developed.

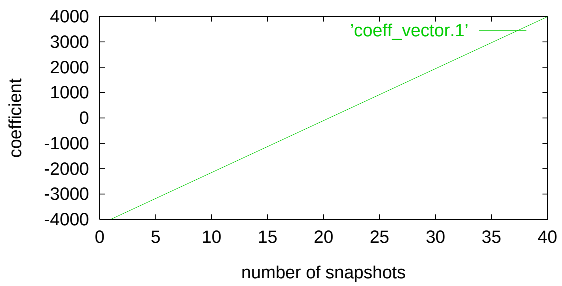
The work presented in this paper lays down the foundation for developing a robust and unified condition monitoring tool using the Karhunen-Loève transform. In this



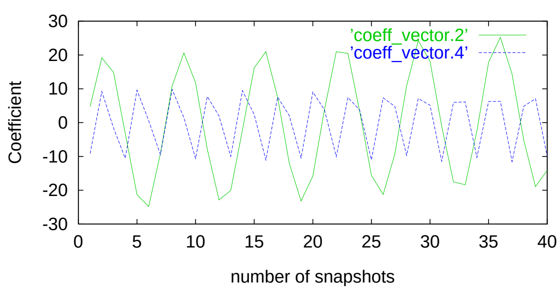
(a) Eigenvectors #1 and #2.



(b) Eigenvector #4.



(c) Coefficient Vector #1.



(d) Coefficient Vector #2 and #4.

Figure 4. Simulation Results: Multicomponent Signals

paper, simple inputs are used to understand the extension of the KL transform to condition monitoring. The significance of the results are discussed in the context of condition monitoring. Numerical results are then presented to demonstrate the power of the KL transform in decomposing multi-component and high-dimensional signals into individual fault features for monitoring.

We aim to use these preliminary results to further develop the mathematical foundation of the method and present the field with an exciting new tool for condition monitoring. In addition, experimentation using surface profiles from grinding, milling, and Selective Laser Sintering processes is being planned (Tumer et al., 1995; Tumer et al., 1997a). Finally, further simulations exploring various manufacturing signals, including nonstationarities, are also being planned (Tumer et al., 1997b). The culmination of these various aspects of our research will result in a powerful fault detection and condition monitoring tool for manufacturing and design.

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