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A FIRST CLASS OF COMPUTATIONAL TOOLS FOR PRELIMINARY ENGINEERING DESIGN

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Abstract

Preliminary engineering design intrinsically consists of imprecise descriptions of the input parameters. We present new conceptual and algorithmic procedures for dealing with such imprecise descriptions. Specifically, a two-part method is outlined for performing design calculations on these "fuzzy" parameters, as well as determining a measure for the parameters' coupling. By interpreting the input set of variables for preliminary design in terms of fuzzy sets, we demonstrate how the engineer may associate his subjective meaning with the input parameters and the output functional requirement, leading to the foundation of our approach. An example of the method highlights the primary issues and the implementation scheme as a computational tool.

1 Introduction

Research efforts in preliminary engineering design have increased significantly in recent years, especially with regard to computation. In fact, a previous paper [5] describes the design process mathematically such that certain algorithmic (computational) tools become apparent to aid the engineer in all facets of preliminary design. One of the first class of such tools is an efficient means for *carrying out design calculations with imprecise parameters.* An approach to meet this objective will be presented in this paper, along with its implementation scheme.

2 The Approach

A designer would prefer *not* to fix parameters in the early stages of the iterative problem solving process, but rather to evaluate imprecisely described design alternatives. We may readily understand this preference when considering the performance parameters of a design as the outputs of engineering equations. For example, a form of Newton's second law may be used within a design problem such that a requirement on force must be met. Force then is the performance parameter; mass and acceleration are the imprecise input parameters. The authors have developed a two-part method for meeting this need of the designer: (1) a one-pass scheme which calculates all possible output quantities from the imprecise inputs, and (2) a mathematical measure which identifies the importance of the inputs with respect to the output, the performance parameter.

2.1 Fuzzy Set Foundation

In our approach, fuzzy set theory (fuzzy sets and fuzzy arithmetic) forms the basis for interpreting the imprecise input parameters in a design, as well as the output result. A fuzzy set (as developed by Zadeh [6]) may be understood as a set with boundaries which are not well defined and with the characteristic of a continuum of grades of membership. Representing a given input design parameter by a fuzzy set, meaning may be derived directly from the associated membership values according to the desires of the designer. The more confident, or the more the designer desires or expects to use an input value, the higher its membership in the parameter's set. More specifically, the parameter's values with membership of one (1) are those that the designer rates the highest in terms of confidence; whereas, parameter values with memberships of zero (0) are those he feels sure can be excluded from consideration.

Just as the input design parameters may have fuzzy sets (meaning) associated with them, the output of the design calculations (a fuzzy set), also has a direct interpretation as follows: the value(s) of the set which take on membership value(s) of one (1) may be directly traced back to the input parameter values which have membership of one (1). Other output values with a membership value (x) which is less than one (1) may be similarly related to at least one input parameter's membership value equal to (x), the other parameters' memberships greater than or equal to (x). If the resulting output value for the membership equal to one (1) satisfies the performance criteria in the perfor-

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mance space of the design, then the design may proceed without further detailed numeric analysis at this preliminary stage of the design, allowing the engineer to concentrate on other functional requirements which require more precision. However, if the performance criteria are not satisfied, then the engineer can examine the output membership function to determine where the acceptable output value falls. The inputs parameters may then be easily and directly traced for all values on the characteristic curve itself, providing the engineer with information on the contribution of each input parameter to the output value. This method permits a designer to first proceed forward through a calculation to reach a (fuzzy) output. Then, if he decides a different output value is required, proceed backwards through the calculation to arrive at the the values for the required inputs. This backwards path is a natural consequence of fuzzy arithmetic implementation developed by the authors, and requires no further calculations once the forward path has been calculated.

2.2 Functional Shapes of Design Parameters

An engineer (subjectively) assigns a most desired (most confident) value or interval to every input parameter for a performance parameter. Values (greatest and least) of zero confidence for eacb parameter are also selected, defining a range of acceptable values. Thus, the results are presented in the form of a triangle (single most confident value with linear interpolation to the zero confidence values) or a trapezoid (interval of most confident values). For preliminary design, the experiments conducted to-date indicate that these two classes of fuzzy set shape will provide sufficient information and accuracy. These types of functions also satisfy the normality and convexity conditions required of fuzzy numbers. If it proves necessary to provide higher order functions, they can be included without modification to the overall concept and process described here.

Although triangular and trapezoidal functions may adeqnately represent the inputs, tbe outputs of design performance analysis will very rarely be symmetric-linear functions. In fact, a fuzzy multiplication with triangular input functions does not result in a triangular output function, but instead two combined functions raised to the one-half power. Addition and subtraction will preserve the overall shape of the input function (e.g., a triangular input will merely be attenuated by addition or subtraction), but division will instead cause similar results as the multiplication operator. In general, highly sloping curves of different shape than the input may be expectcd for fuzzy computations. This does not detract from the meaning of the output; the result of a fuzzy calculation may be easily interpreted as previously discussed, whatever its shape.

Overall then, fuzzy sets and the associated meaning ascribed to the inputs and output provide an appropriate means for handling computations with imprecise parameters in engineering design, where the specified design inputs may be represented by triangular and trapezoidal functions.

2.3 FWA Algorithm in Design

Kaufmann and Gupta [2] present analytical methods for calculating a fuzzy output from fuzzy inputs. Although the method is straight-forward in its approach, the manipulation of symbols and the solution of expressions which include high order polynomials, both in the numerator and denominator, make this method infeasible for computer-assisted design applications. A discrete numerical approach is therefore necessary to meet the computational requirements for handling many design parameters.

Many discrete and analytical methods exist for carrying out extended operations with fuzzy sets (or fuzzy numbers). However, the Fuzzy Weighted Average (FWA) algorithm as presented by W.M. Dong and F.S. Wong in [1] outlines a simple and efficient algorithm which meets the requirements for carrying out engineering design calculations. A new, highly computationally efficient implementation of Dong and Wong's algorithm [1] has been developed by the authors. Using this implementation version, reasonable memory management and complexity for the method may be obtained. Specifically, the actual complexity of the algorithm is of order

$$H \sim M \cdot 2^{N-1} \cdot \kappa$$

where H equals the number of operations, N is the number of input parameters, and κ equals the number of multiplications and divisions in the routine.

2.4 Identifying the Role of Input Parameters

The approach to this point has described a method for discretely calculating the possible output quantities from imprecise input parameters. This section will describe a means of determining the qualitative relationship between input parameters and a given (output) performance parameter. In any design, some input parameters are very strongly coupled to the outputs, and others are nearly independent. A mathematical measure will provide the means for imprecisely determining this coupling.

We have developed a new measure which we will call a *γ-level measure*. We define this measure in the following manner:

$$D(\tilde{C}) = \sum_{i=1}^{|X|} (e^{\beta(x_i)} - 1)^m, \qquad (1)$$

where

$$\beta(x_i) = \begin{cases} \frac{\mu_{\tilde{C}}(x_i)}{\gamma} & \text{if } \mu_{\tilde{C}} \leq \gamma \\ \frac{2\gamma - \mu_{\tilde{C}}(x_i)}{\gamma} & \text{if } \mu_{\tilde{C}} \geq \gamma, \end{cases}$$
$$0 < \gamma \leq 1,$$

and m is an integer such that as m increases, the measure becomes more concentrated for values about $\mu_{\tilde{C}} = \gamma$. The value of γ may be set so that $D(\tilde{C})$ measures values in the support centered about it.

Letting $\gamma = 1$ in Equation 1, we may apply the measure as defined above to the input parameters as well as the result of the computation (the performance parameter), then the results express the qualitative contribution to the output by each of the fuzzy inputs. For the engineer who utilizes fuzzy sets in the description of design and performance parameters, this new measure provides him with the ability to determine some information on the coupling between the inputs and outputs of imprecise calculations. Thns, if the measnre indicates that certain input parameters contribute very little to the analysis, they may be fixed to the most-desired value by the engineer, resulting in a simplification of the design problem.

3 An Example

In order to clarify the ideas presented in the previous section, a simple mechanical design example will be given. The problem is to design a mechanical part which will not fail when subjected to an imprecisely specified transverse load, F. Assuming

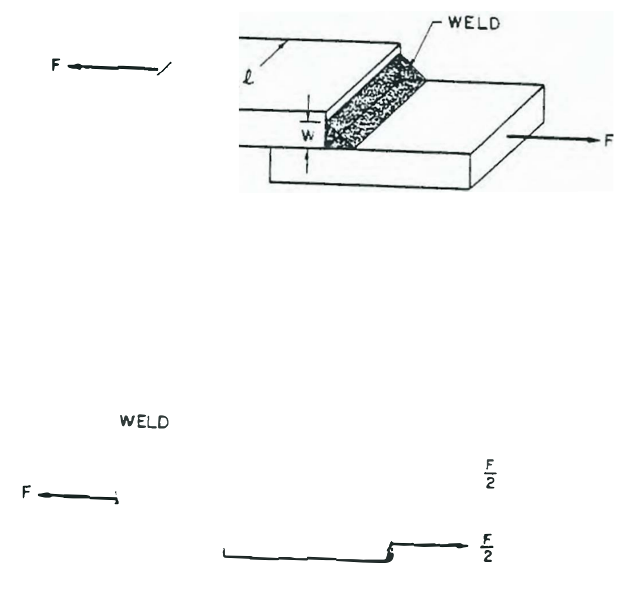

Figure 1: Mechanical Part.

one possible configuration and description of the part has been determined previously (see Figure 1), one obvious performance parameter (i.e., functional requirement equation) for the design is the average stress in the welds of the part. In equation form, this performance parameter may be expressed as:

$$\tau = \frac{F}{wl}, \quad (2)$$

where

$F =$ *applied force*,
$w =$ *width of weld throat*,
$l =$ *weld length.*

The engineer now specifies the input parameters as fuzzy sets according to the procedures found in the approach section. Exemplary input parameters for $\tilde{F}$, $\tilde{w}$, and $\tilde{l}$ may be found in Figures 2 and 3. Using the FWA algorithm and its implementation scheme as described previously, the output $\tilde{\tau}$ may be determined as in Figure 4, where the *tilde* denotes a fuzzy set. If the value of $\tilde{\tau}$ where $\mu_{\tilde{\tau}} = 1$ satisfies the performance criteria in the performance space of the design (i.e., this $\tilde{\tau}$ is within some limits specified by the engineer), the design may proceed without further detailed numeric analysis, allowing the engineer to concentrate on other performance parameters. However, if the performance criteria are not satisfied, then the engineer must examine the output membership function to determine where the acceptable allowable stress falls. After choosing this stress(s), our implementation scheme provides the engineer with a simple utility for

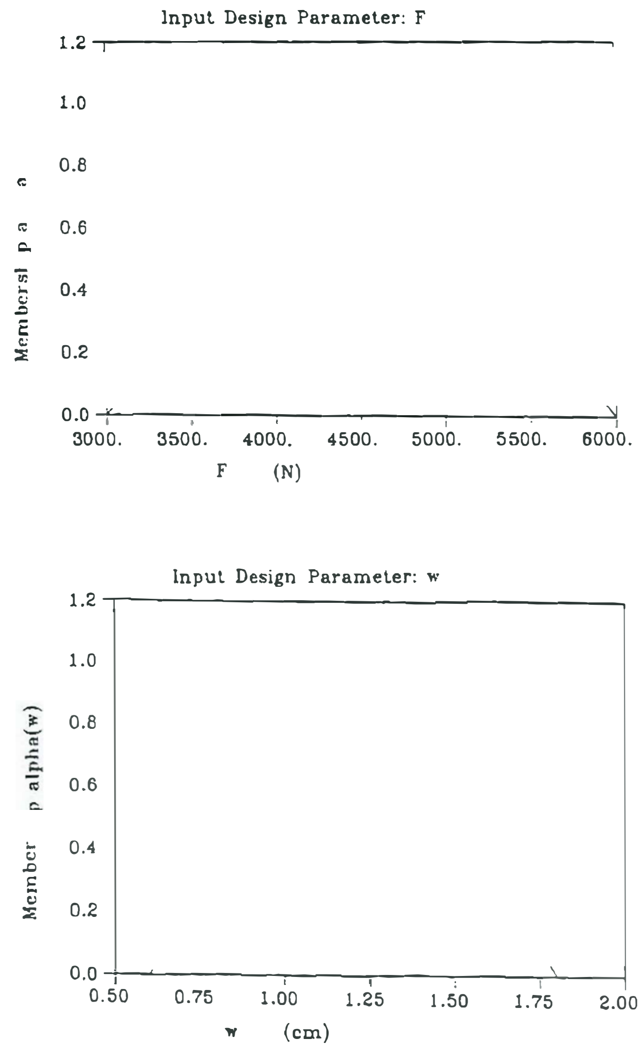

Figure 2: Input Parameters, F and w.

tracing the values of the inputs which contributed to that output stress such that *no further calculations are performed.*

Along with obtaining the specific input values, the *γ-level* measure may be used to provide the engineer with qualitative information concerning the role of the input parameters in the design. For the performance parameter $\tilde{\tau}$, the process to use the measure consists of the following steps:

1. From the support of $\tilde{\tau}$, let λ_1 and λ_2 be equal to the two $\tilde{\tau}$ values for which $\mu_{\tilde{\tau}} = min$ on both the left and right extremes of $\tilde{\tau}$. $\Lambda = [\lambda_1, \lambda_2]$ makes up an interval of the support of $\tilde{\tau}$.

2. Discretize the interval Λ into n equally spaced steps, such that $| X | = n$ in Equation 1.

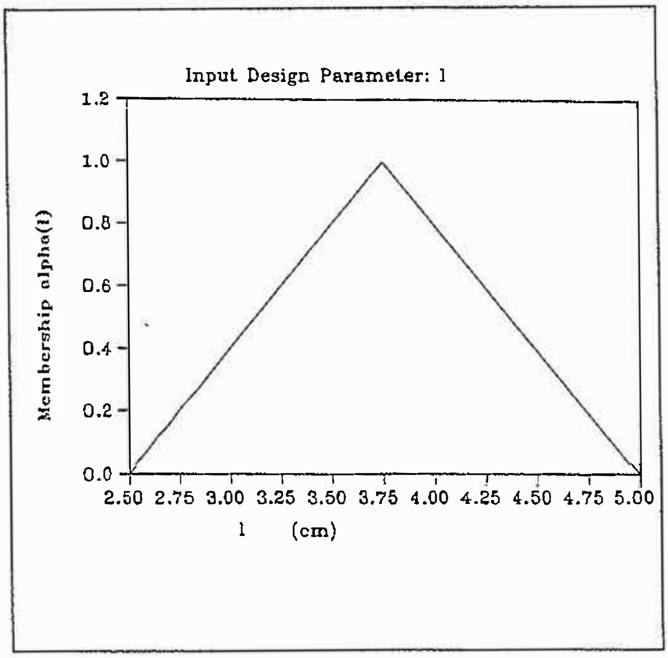

Figure 3: Input Parameter, l.

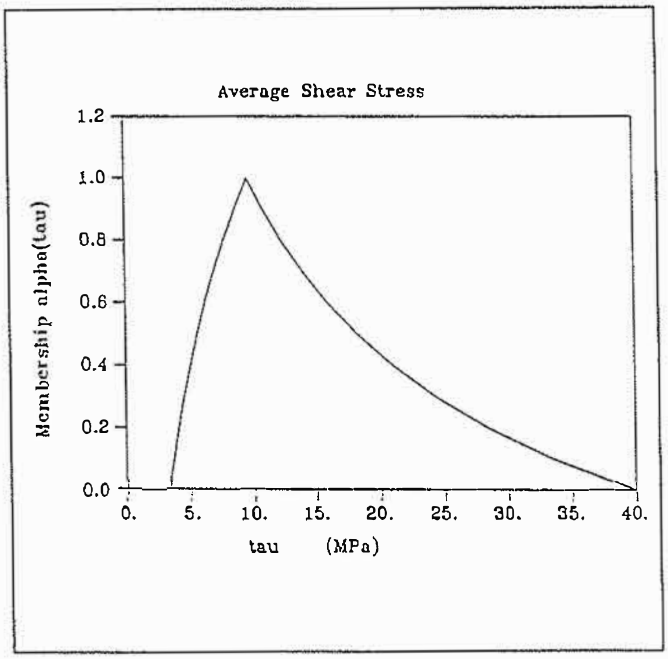

Figure 4: Output Set, τ.

3. For each input parameter, F, w, l (denoted by $\tilde{C}_i$), let all other $\tilde{C}_j, i \neq j$, be set to their crisp value where $\mu_{\tilde{C}} = 1$. For $i = 1, \ldots, 3$, use FWA to calculate the output, Θ_i, where the i^{th} fuzzy input remains fuzzy in the calculation, and all others are made crisp as above.

4. Calculate the *γ-level* measure ($\gamma = 1$) for $\tilde{\tau}$ and all Θ_i.

5. The measures $D(\Theta_i)$ may be compared or normalized with respect to $D(\tilde{\tau})$. The result is an ordering of the inputs according to importance (relative measure), giving a qualitative relationship of inputs to the output.

4 Conclusions

The technique and implementation developed here represents a new, and powerful approach to manipulate imprecise design data. Although the example shown here is simple in nature, it may be inferred that our approach can be applied to a wide range of engineering design problems, and provide the ability to perform design calculations on a variety of imprecise parameters. The (correspondingly) imprecise calculation results provide far more information to the designer than either conventional, single valued analysis, or interval analysis.

Additional useful information that this method can provide, through the use of the *γ-level measure*, is the coupling between design parameters (inputs) and the performance results. This coupling information can be used to focus the engineer's resources on those aspects of the design problem with the largest effect on the resulting performance.

For a more exhaustive treatment of the ideas and concepts of this paper along with further examples, the reader is referred to [4] and [3].

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