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In fact there are a myriad of theoretical issues, which need to be addressed and these are explained in the following section.

5 Theoretical Issues in Fractal Parameter Estimation

In the development of a fractal-based approach for tolerancing, it is imperative to characterize the error structure in manufacturing and the tolerance information for design in an unambiguous and uniform fashion. A number of definitions for fractal dimension exist and each definition is based on an underlying algorithm, which captures some spatial/spectral property, e.g., length of profile, area of surface [11]. One of the major problems with traditional tolerancing methods is that different measurement techniques and data fitting algorithms give different measurement results [30], and a similar threat looms for the fractal-based tolerancing method being developed. This necessitates the development of an unique definition for the fractal dimension and other related parameters. The modified box counting algorithm used in [28] is based on the physical analogue of the inspection process; however, the concept and mathematical foundation has to be refined further so as to be useful in the design stages also.

In addition to developing a suitable algorithm, the reliability of the computational estimate must be ensured, by standardizing some measurement parameters, which influence the dimension estimates, e.g., the number of points in the profile, data sampling rate and so on [2, 5]. In fact, some of these problems are generic even to conventional tolerance specification and verification methods, and have only elicited some recent, preliminary solutions [18]. The idea of a *multifractal* model for a full characterization of the surface, including both tolerance and surface roughness ranges also depends on this standardization. A detailed study of various algorithms and the problems in estimating them is planned with a view to consolidating the theoretical issues underlying fractal-based tolerances.

6 Conclusions

This paper traces the progress of fractal-based tolerancing methods proposed by the authors. A brief description of the parent discipline, DFM, emphasizes the role of tolerances in the information flow between design and manufacturing. A critique of current tolerancing practices, followed by a set-theoretic description of design, serve to highlight the potential of fractal-based methods in abstracting manufacturing information. This abstraction leads to the definition of the fabrication function, a prospective design tool, aiding more accurate performance and manufacturability assessment.

Against this backdrop, the project status is reported, followed by a detailed description of the more recent research directions, i.e., modeling manufacturing processes, developing computational methods for fractal-based tolerance design, and performing physical experiments to verify the fractal structure of manufacturing errors in tolerance scales. The errors from a milling process conform to the proposed fractal structure. Future goals have been outlined in terms of theoretical issues, modeling extensions, further development of computational tools, and experiments.

Figure 8: Profiles from the first replicate.

Test ID.	Random Order	FACTORS			Error	
		s	f	d	Δh μm	D_f
a	1	−	−	−	75	1.036
b	8	−	−	+	40	1.019
c	5	−	+	−	90	1.048
d	7	−	+	+	89	1.049
e	3	+	−	−	65	1.007
f	4	+	−	+	51	1.012
g	6	+	+	+	57	1.037
h	2	+	+	+	88	1.047

Table 2: Layout and results for 2^3 factorial experiment.

4.3.3 Results and Discussion

The first experimental replicate has been machined, and the height variations along a linear element of the surface have been recorded using a digital electronic indicator. The modified box-counting algorithm described in [28] is used to estimate the fractal dimension. The conventional error parameter, viz., the maximum peak-to-valley height Δh is also calculated for comparison purposes. The matrix of the tests, and the experimental results, is given in Table 2. The plots of the profile traces, obtained from the above experiments, are also depicted in Figure 8. Each profile is labeled with the corresponding "Test ID." from Table 2. These results are discussed in the next section.

4.3.4 Exposition of the Results

The above experimental results *firmly substantiate the fractal nature of manufacturing errors.* The calculated fractal dimensions have an absolute correlation coefficient greater than 99% in all cases. Fractal dimensions exhibit the capability of capturing the total variability information as opposed to the maximum peak to valley error. However, the box-counting algorithm used in these experiments is unable to detect the slope error in the profile (e.g., Test ID: e). This drawback has been recognized and addressed in [28]; the current physical evidence mandates the identification of additional parameters/algorithms for a complete characterization of the actual profile.

While a definitive conclusion about the influence of machining parameters on the machining error must await the measurement of the second replicate profiles, the detrimental effect of higher feedrate is already apparent; studies on surface finish have revealed a similar trend [3]. Such inferences only partially satisfy the first objective of the experiments. A more detailed study of the individual factor effects and the interaction effects is planned after the data collection for the second replicate. Additional experiments investigating the ability of fractal-based parameters to distinguish between roughing, and finishing processes are also being planned. While this segment of the experiments is devoted to rectangular workpieces and straightness errors, additional experiments to complement the dynamic modeling of the turning process (Section 4.1) for circularity errors are being designed. Concurrent research to establish a firm theoretical basis for the consistent interpretation of the above results and the development of suitable models is also being pursued.

Figure 7: Flycutting on a Milling Machine.

Level/Factor	$s(fpm)$	$f(in./rev.)$	$d(in.)$
$+$	300	0.025	0.02
$-$	100	0.010	0.01

Table 1: Factors and Levels for Milling Experiments.

method of experimentation is incapable of detecting interaction effects and also requires significantly more experimental runs. Hence in the current studies, a more economical and robust factorial design approach is adopted [22].

The flycutting operation is performed on a Bridegeport vertical milling machine, using a HSS cutter. The workpiece material is Aluminum 2024; no cutting fluid is used in the experiments. The variable factors and the levels chosen for this particular study are described in the next section.

4.3.2 Factors and Levels

Three factors are considered in this study: the cutting speed s, feed f, and depth of cut d. Each factor is considered at two levels. This constitutes a 2^3 factorial design [22]. The values selected for the factors are presented in Table 1 in appropriate units, where $+$ indicates the higher level and $-$ indicates the lower level of the corresponding factor. The values shown are modified versions of the values recommended by the Machinability Data Handbook [1], taking into consideration the power and stability of the machine tool selected for the experiment. Each combination of levels is replicated to estimate the experimental error. In addition, the order of the eight experiments is randomized in each run, to counteract unknown variations [22].

menu-driven queries of these spatial relationships. An example query selected from menu options reads "Display all orthogonal structural primitives (to a selected primitive)." Operations such as this query are of use to computer-aided manufacturing programs, such as automatic process planning systems. The final aspect of the feature-based modeling system is the generation of a semantic network. The semantic network is the data structure (undirected graph) supporting the set of structural primitives and their spatial relationships. Other computer-aided design systems may access part information through the semantic network.

Development of the fractal-based tolerance representation within the test bed consists of a fractal surface generation routine for 2-D fractal Brownian motion [21]. This routine creates a topologically complete geometric object, in accordance with the manifold geometric requirements of the test bed. Most geometric modeling systems provide a finite number of geometric objects for creating more complex objects and assemblies. The test bed provides routines for user definable objects, in this case a fractal surface primitive. Such objects provide the capability of performing Boolean operations with fractal surfaces for applications such as clearance between the structural primitives of two features.

Future directions for the fractal-based tolerance representation within this research component include: the comparison between theoretical and physical or modeled data, and applying fractal surfaces on arbitrarily shaped objects. The approach for these issues is investigating variations of the offset solid theory proposed by Requicha [23] and investigating mapping methods, such as affine transformations. Using the offset solid theory, physical or modeled part representations are checked "in tolerance" through boolean operations between the part and tolerance zone. One proposed variation on the offset solid theory is using non-manifold geometry to check independent tolerance constraints. In such a variation, for example, two concentric tubes check a cylindricity tolerance of a shaft without respect for the flatness tolerance of the shaft top. A non-manifold approach, with fractal tolerances, provides the capability of treating the shaft side primitive independently from the shaft top primitives. Such an approach also adheres to the total variability concept discussed above, in that fractal tolerances are used instead of idealized peak-to-valley tolerance zones.

4.3 Experimental Work

An important component of this research project is to verify the theoretical and modeling results with physical experimentation. The first process selected for such a study is the face milling operation, where the cutting is carried out by means of a flycutter. This choice is motivated by the desire to pursue the characterization of straightness errors studied in the earlier phases of the project, and the availability of machine tools. A flycutter is a single point tool, mounted in a suitable tool holder, which in turn is mounted in the spindle of a vertical milling machine. It is used to machine light cuts on flat workpieces (Figure 7). The objectives of the study are twofold: first, to study the existence of fractal-based error patterns in the workpieces, thereby verifying the simulation results, and second, to determine the effect of cutting parameters on the above patterns, if they exist. The design of experiments to attain these objectives is described in the following section.

4.3.1 Experimental Design and Conditions

The simulation experiments described in [28] are "one-at-a-time" experiments, in that only one of the machining variables or factors is changed, keeping the other two factors constant. This

Figure 6: Example Test Bed Features and Structural Primitives.

part and assembly models, testing experimental and/or modeled data with respect to the tolerance theory, and exporting such information to other computer-aided engineering software. The term suitable refers to the degree of accessibility of the test bed program by researchers. A well designed test bed may be customized for different theory alternatives. This is especially important during the development phase of the fractal-based tolerance theory.

4.2.3 Current Progress

Research by da Silva, Wood, and Beaman [9] forms the basis of the feature modeling interface to Purdue University's TWIN geometric modeler. da Silva, et al.'s research investigates features and their spatial relationships for use by a geometric reasoning system for mechanical design. This involves a more "informationally complete" model of mechanical parts, which includes features, the primitives which comprise features, and the geometric relationships between these primitives. One result of this research is an "intermediate geometry" object representation, encoding geometric feature information along with associated spatial relationships. Intermediate geometry represents parts and assemblies in terms of structural and geometric primitives. Structural primitives are the faces comprising a part, such as tops, bottoms, sides, and ends. Geometric primitives represent the spatial relationships between the structural primitives, such as parallel, orthogonal, and planar. This object definition level is application independent [9]. The test bed directly converts between the TWIN B-rep of an object and this more robust intermediate geometry representation. Figure 6 illustrates example features supported by the test bed and their structural primitives.

The feature-based modeling system consists of stock and feature definitions, structural primitive recognition, view and display options, and spatial relationship representation. As users generate features on stock parts, the test bed determines the structural primitives associated with each feature. For example, the test bed detects the tops, bottoms, sides, and ends of a generic feature, and determines the spatial relationships between structural primitives. The test bed also allows

4.2 A Computational Fractal-Based Tolerance Representation

This section describes the implementation of a fractal-based tolerance representation as part of a geometric modeling system. Work thus far consists of a feature-based geometric modeling test bed, incorporating fractal-based surface generation algorithms. The motivation, design issues, approach, and current progress of this research component are presented.

4.2.1 Motivation

The primary motivation in developing a computational fratcal-based tolerance representation is to refine the overall fractal-based tolerancing theory. A computational method for representing fractal-based tolerances benefits the development of this tolerance theory in several respects. Upon creating geometric models of parts and assemblies, a computational representation provides a method for comparison of physical, modeled, and expected tolerance data. Such an implementation validates the practicality of a fractal-based tolerance representation in modern mechanical design environments. This project also serves as a graphical test bed for computational geometry research at the University of Texas at Austin.

The test bed consists of a feature-based geometric modeling system and fractal-based profile generation algorithms. The feature-based geometric modeling system serves as the groundwork for the fractal-based tolerance representation. This system provides an intuitive interface for creating parts and assemblies. A user generates parts and assemblies by stock removal and addition from a feature library containing such objects as holes, slots, and pockets. The feature-based modeling system is based on research by da Silva, Wood, and Beaman [9] and incorporates an "intermediate geometry" object representation. Intermediate geometry represents parts as collections of structural primitives, or faces. This representation also protects the fractal-based tolerance representation and test bed from dependence on any specific geometric modeling system. An internal intermediate geometry representation for the test bed insures greater transportability of the tolerance representation between geometric modeling systems.

The fractal-based tolerance theory proposes to represent tolerances with an unambiguous, minimal set of parameters, namely, but perhaps not exclusively, the fractal dimension. Fractal-based surface generation routines are necessary to convert the fractal-based tolerance theory to usable geometric modeling data. Feature-based modelers allow these parameters, such as the fractal dimension, to be attached as attributes to a feature, as opposed to the complex surface information. When actual surfaces are needed in the geometric model, e.g., during boolean operations with other features, surface generation methods transform the attributes to valid surface data. Using fractal-based surface generation methods in such a representation scheme releases geometric modeling programs from the additional computations necessary for fractal object transformations. The following section discusses the issues concerning the computational fractal-based tolerancing theory and the approach taken to address these issues.

4.2.2 Design Issues and Approach

Current trends in mechanical design affirm the applicability of geometric modeling systems in design environments; however, there exist deficiencies in explicitly representing and operating with tolerances [38]. The first step in addressing this deficiency is the development of a test bed to aid in fractal-based theory development. The test bed provides a suitable platform for creating geometric with stiffness and internal damping elements. The feed velocity flows through the carriage inertia and damping elements, which constitute a common flow junction. The derivative causality on the inertia element in the x direction causes the inertia element to be dependent in the x direction. The flow through the carriage stiffness element is dictated by the input feed velocity minus the output tool velocity in the x direction, implying a common effort junction. The cutting tool is modeled as consisting of inertia, stiffness and internal damping elements, with a common velocity, implying once again, a common flow junction.

The bottom section of the turning system also includes a translatory motion in the y direction, assigned in the cutting depth direction of the tool. In this model, it is proposed that oscillations in the translatory direction, along with the rotary direction, will cause the entire system to have a variable motion in the y direction prescribed. For instance, the carriage assembly will have inertia, stiffness, and damping elements in both the x and y directions. Similarly, the cutting tool, even though it is dictated to move in the x direction only, will also have an oscillatory motion in the y direction. The two directions are combined by introducing a two-port inertance field corresponding to the carriage assembly motion in both directions. The inertia component in the y direction results in an integral causality. The flow through the y component of the elements mentioned is distributed in the same manner as in the x direction.

The tool contact point incorporates the forces generated during the cutting process due to resistance at the tool/workpiece contact point and the contact stiffness due to elastic deformation at the contact point. When combined with the rotational and translational components, the tool contact point model provides a complete model of the turning system. The following section presents the simulation approach to be adopted in this component of the project.

Simulations The overall objective of this project component is to simulate the bond graph model to determine the performance of the turning process. In particular, the variable of interest is the total variation in the depth of cut direction y that affects the tolerances and surface finish of a machined part. The model simulation must accurately describe the turning system based on the system physics. The outputs of interest in the model will be compared to actual turned parts to determine the accuracy and the correctness of the model. In order to assure the performance of each system component, the simulations are carried out separately for the rotational system, the translational system, and the tool contact point. First the rotating system, then the translational system must perform adequately before combining the two sections with the tool contact point bond graph. Once each significant section of the system performs as expected, the entire system bond graph will be simulated.

Currently, the simulations for the rotational components behave as expected. The current focus in this research component is to obtain satisfactory results for the translational section of the turning system, and then for the entire bond graph. A sensitivity analysis will follow these simulations for realistic ranges of design and cutting parameters. The long term goals include analyzing the surface variation obtained from the model, computing its fractal dimension, and comparing the results to actual surface profiles and fractal surface representations. After completing the models, simulations, and experiments for the turning process, other machining processes will be studied, following the methodology currently adopted.

Figure 5: Bond Graph Model of the Engine Lathe Turning System.

Bond Graph Model In order to study the dynamics of a complex system, a model of the system is typically developed. A model is a simple and abstract construct used to predict system behavior [15]. System modeling is typically defined by two goals [6]. The first goal aims at representing all physically realizable systems. The second goal requires that all possible system models correspond to realizable physical systems [6]. A modeling technique called bond graph modeling is capable of achieving both goals systematically [6]. Bond graphs, based on energy and information flow, provide a uniform notation to construct system models for all types of physical systems, allowing one to study the structure of a system model. A bond graph simply consists of subsystems connected by ports which represent power bonds [15]. Energy leaving one subsystem is instantaneously received by the next subsystem. The models are translated into differential equations or computer simulation schemes algorithmically.

A basic knowledge of bond graph modeling is assumed in this paper, and the reader is referred to Karnopp et al. [15] for more detail. As a brief review, a bond graph, along with the constitutive relationships and the connecting equations at the junctions, provide a set of state equations, i.e., a set of first order differential equations. The variables in these state equations are called state variables. The number of independent energy storage elements is determined by the number stiffness and inertia elements that result in integral causality. The momentum and displacement variables associated with these independent storage elements define the state variables. Connecting equations, obtained by the sum of the power variables at the junctions, are used in writing state equations, along with the constitutive laws. Common flow in the system is designated by a 1-junction, and common effort by a 0-junction. The C, I, and B elements are the compliance, inertia and internal damping respectively. The energy storing elements (C and I) and the dissipative element (B) each have constitutive relations that will be used to expand the connecting equations.

Based on the system component description and the bond graph modeling technique, a bond graph model is developed for the turning system, as shown in Figure 5. In order to more accurately analyze and simulate the turning process model, the system is divided into three major sections: rotational components, translational components, and the tool contact point. The rotating portion of the turning system includes the spindle, the chuck, and the workpiece to be machined. Currently, the resistive elements due to the electric motor resistance, bearing friction, and friction between components are neglected for simplicity. The spindle is modeled as having mainly torsional stiffness. The rotary inertia element of the spindle results in derivative causality, i.e., dependent element state. The spindle is also assumed to have an internal damping. The torsional stiffness and the internal damping elements share the same angular velocity, dictated by the input spindle angular speed minus the output chuck inertia angular speed. Common velocity implies a common flow junction, and the sum of the input and output speeds implies a common effort junction. The torque at this common effort junction is dictated by the sum of the torques provided by the capacitance and damping elements. The capacitance due to the spindle torsional stiffness results in an integral causality, which assures an independent energy storing element. The chuck, located between the spindle and the workpiece, follows the same flow and effort distribution scheme. The only difference is the chuck inertia element which results in an integral causality, thus producing another independent element to the system. The workpiece shares flow and effort in the same manner as the chuck.

The translating components include the carriage assembly (carriage, cross slide, and tool post), and the cutting tool. Once again the friction between components and friction due to gearing and the rack and pinion are neglected for simplicity. The carriage has mainly an inertia element, along

Figure 4: Schematic of an Engine Lathe.

middle of the figure are the rotational and the translational sections of the turning system, which are placed between the headstock and the tailstock for support. The upper portion is the rotational section consisting of the spindle, chuck, and the workpiece. The lower translational portion consists of the carriage, cross slide, the tool post, and the cutting tool, guided on ways and a feed rod. The entire system is supported on a bed or foundation. All of the components of the turning lathe are assumed to contribute to the machining process. Therefore, it is necessary to describe their physical function in detail.

The primary motion in the system is the rotation of the workpiece, produced by the movement of a series of gears driving the main spindle. The gears are driven by an electric motor. The main spindle and the gears are mounted in the headstock. The single-point cutting tool is held in a toolholder (or toolpost), which is mounted to a cross slide on the carriage. The translatory movement of the carriage is the feed motion. The carriage is driven along the bed by a lead screw or a rack, pinion gear, and feed rod assembly. The lead screw and feed rod are connected to the main spindle through a gear train. Relatively "long" workpieces are supported at their ends with a center held in the tailstock. The toolpost is mounted on a cross slide, providing the radial movement. The cross slide is guided in the carriage, which in turn receives support from the ways machined in the bed, ensuring rigidity and isolation from vibrations [4, 26]. Based on this description of the engine lathe, a schematic of the turning system is shown in Figure 4. In the following section, this physical description or schematic of the turning system is transformed into an abstract graphical representation, known as a bond graph model.

Figure 3: Detailed Engine Lathe.

Figure 2: Lathe Turning.

creating a more realistic and comprehensive model of the turning process, as a start, with the goal of advancing the theory of fractal-based tolerancing. As opposed to the model presented in [28], actual cutting processes are more complex. The modeling technique used in this work provides a more complete and robust model, capable of accounting for more of the complexity of machining processes. The following section presents the current results achieved in this research component.

4.1.3 Results

Turning Process The most widely used cutting and machining operations are: turning, milling, and drilling [27]. In this project, the focus is on turning. Turning is a process using a single-point tool that removes material to produce a surface of revolution. The machine tool is usually a lathe [27]. Most commonly, the workpiece material is held in a chuck on a lathe and rotated; the tool is held rigidly in a toolpost and moved at a constant rate parallel to the axis of the workpiece, removing layers of material to form a cylinder or a surface [31]. Figure 2 shows a turning lathe with its primary process parameters. These parameters are defined as follows:

- V: cutting speed – rate at which the uncut workpiece surface passes the cutting edge of the tool;
- f: feed – the distance moved by the tool in an axial direction for each revolution of the work;
- w: depth of cut – thickness of material removed from the workpiece, measured in the radial direction.

Global System Description Before analyzing the surface profiles, it is necessary to understand the physics involved in a turning system. A clear understanding of system physics constitutes the basis for an adequate bond graph model. Figure 3 shows an engine lathe in detail. On the left hand side of this figure is the headstock and on the right hand side is the tailstock. Shown in the

- Development of more comprehensive dynamic models of manufacturing processes, with the goal of characterizing process errors in terms of fractal parameters.
- Exploration of computational methods for incorporating fractal-based tolerances in geometric models of mechanical components. An intermediate step is the development of a suitable test-bed for integrating the geometry of a component in terms of features, with fractal-based surface generation algorithms.
- Verification of simulation results from previous studies by means of physical experiments, which involves experiment design, machining of surfaces, and measurement and analysis of the machined profiles to detect possible fractal structure.

Each sub-problem is examined in detail below, in terms of the background and motivation, the approach adopted to address the problem, and the outcome from the investigation.

4.1 Modeling of Manufacturing Processes

This section addresses the research topic involving models of machining processes. First, the motivation for the topic is discussed, followed by the approach adopted, and finally the results obtained at this point in the project.

4.1.1 Motivation

The primary objective of this research component is to study the mechanics of manufacturing processes, develop a model for a machined surface profile, and simulate this profile. These simulations will be compared to the results from a fractal representation and experimental data. In addition, the project's fractal methods will be used for determining the structure of the surface irregularities predicted from the models. Based on these comparisons, the models will be evolved to capture the major parameters of the machining system and the major large-magnitude effects that contribute to the surface irregularity of machined parts. Ultimately, the program is aimed at determining a process model for parameter interactions and their influence on tolerances.

Surface models represent the first step towards describing a machined surface profile by means of fractal methods. The ongoing research program is aimed at finding a structured method of assigning tolerances according to the machining process used and the cutting and design parameters employed. To estimate the structure of the generated surface profile, it is proposed to model the machining processes, currently turning, and analyze the resulting surface profile. The modeling of the machining processes will include prominent large-magnitude machining errors for a more realistic representation of the machined surface profile. The analysis of the surface profile will reveal an alternative method of representing and specifying tolerances. Modeling is accomplished by bond-graph modeling techniques which provide as systematic means of incorporating most error sources for a machining process.

4.1.2 Approach

The preliminary results presented in [28, 37] support the idea of the variational structures produced from manufacturing processes. However, simulations using more realistic models need to be developed to verify these results. The work in this component of the project focuses on by the authors in [37], provides a forum for modeling manufacturing processes, for representing manufacturing errors associated with these processes, and for describing realistic part models in mechanical design.

3.2 Fractal-based Tolerance Representation

The inceptive stage of the project explores fractals and the feasibility of using fractals to characterize tolerances [37]. The problem considered is the representation of roundness tolerances on the shaft of a journal bearing. The errors from two manufacturing processes, i.e., turning and grinding, are generated from stochastic data and superposed on the ideal profile of the journal shaft. A fractal dimension is estimated for the irregular profile using a modified compass method. The resultant fractal dimensions reflect the expected process capabilities of the respective processes. Some preliminary experimental results also exhibit similar trends. A complementary scheme called the Karhunen-Loeve method is introduced conceptually, and the possibility of a multifractal representation for both tolerances and surface roughness is conjectured.

3.3 Structure of Manufacturing Errors

This phase of the research extends the theme of characterizing manufacturing errors by adopting a more detailed investigation into the mechanics of machining [28]. The shearing mechanism of metal removal and typical error sources are incorporated in a model simulating the shaping process. The fractal dimension for the resulting irregular profile (straightness error) is calculated using a modified box-counting algorithm, interpreted as a physical analogue of the actual inspection process. The concept of total variability is introduced, and it is demonstrated, based on the simulation results, that fractal dimensions indeed capture total variability as opposed to the traditional tolerance zones.

3.4 Fractal-Based Tolerances in Design

This segment of the research offers a preliminary perspective of how the fractal-based tolerance information can be useful in engineering design [29]. The design of a thrust bearing is used as a case study. Bearing straightness error, expressed in terms of a fractal dimension, is used in a fractional Brownian motion algorithm to generate a realistic profile for a single bearing pad. The pressure distribution, load carrying capacity, and friction force are evaluated using the relations from hydrodynamic lubrication theory. The errors are also linked to conventional manufacturing processes, and the results confirm the detrimental effect of large deviations from nominal.

The above phases of the research address crucial issues concerning fractal-based tolerances in design for manufacturing. The results have inspired exploration of new avenues. The following section describes these new avenues, the methods adopted to address them, and recent results.

4 New Directions and Results

The impetus from the progressive stages of the research has been channeled to address three important sub-problems:

$\mathcal{H}$ may be formulated in terms of the classical shaft design formulae, where the variations in the interface between the shaft profile and the supporting bearings are captured by the fabrication function. An example acceptability index for the sets of shafts in the context of $\vec{m}$ may simply be the the difference between the clearance of the bearings and the shaft profile variations. If this quantity is positive, the acceptability index is one, otherwise zero.

2.4 Abstraction of the Design and Manufacturing Vectors

The fabrication function which maps the design and manufacturing vectors to the actual product or part model is poorly defined in current practice. The problem is one of information proliferation, where the designer must consider both design and manufacturing information. This information, except in relatively small problems, can become unwieldy. Most designers must therefore resort to the evaluation of ideal geometries, not considering the variational data.

Designers use abstraction to reduce a problem to its most essential level. This is achieved by relaxing constraints, reducing the number of variables and/or concepts that must be considered simultaneously, or by simplifying the model, and hence generalizing it [24, 25]. Fractal-based tolerancing aims at abstracting the multitude of manufacturing variables and their interactions in the form of a minimum parameter set, i.e., the fractal dimension. This is expected to explicitly address the design and manufacturing information problem by facilitating actual part definitions and subsequent analyses.

3 Current Status of the Project

As stated above, the global objective of this work is to represent design for manufacturing information (the fabrication function) in terms of fractal parameters. This section presents the current status of the research toward this objective. The progress in the project is best reported as three different, yet inter-related phases:

- Preliminary study of manufacturing errors to explore feasibility of using fractal-based methods to represent tolerances [37].
- Investigation into the structure of manufacturing errors based on machining mechanics [28].
- Feasibility study of using the proposed fractal-based tolerancing methods in mechanical design [29].

The motivation to adopt a fractal-based approach for tolerancing is briefly described below, followed by a summary discussion of the above topics.

3.1 Motivation

Fractals, developed by Mandelbrot, are used to describe a wide variety of irregular shapes and phenomena [16]. *Physical* fractals [13] provide a flexible tool to model artificial and natural systems. One related application of physical fractals is the characterization of surface finish in terms of fractal dimension [7]. This information is used to assess the "cleanability" of surfaces produced by different material processing techniques. Such applications, in the context of representing irregular geometry, motivate the use of fractals in tolerance representation. This idea, advanced

The acceptability of the part is verified by means of a decision function, defined as follows:

$$\Delta[\vec{p}\,^{p}_{ijk}, \vec{p}_d] \rightarrow A_{ijk} \tag{3}$$

where $\vec{p}_d \in \mathcal{P}$ and A_{ijk} is the merit or acceptability index of the part. The decision function represents, at a high level, a comparison of the predicted performance of an actual part model to the desired performance. At this point, the iterative nature of the design process is apparent, as the designer may terminate or continue the process through further refinement of the design and manufacturing vectors, modification of the design space, etc. The exact details are given in [36].

It is conjectured that the fractal-based tolerancing methods proposed in this research will aid in a more effective definition of the fabrication function Ξ, thereby yielding more realistic product or part models, as opposed to the idealized geometric models presently used for design analysis. Currently, this function is poorly defined, partly because the concepts of design for manufacturing are only recently developing momentum for implementation, and also because a common, unified theory for tolerances has not been developed.

2.3 An Illustrative Design Example

The following design problem is presented to better illustrate the spaces and functions defined above. Consider the design of a shaft to transmit a specified torque. The shaft is supported by journal and thrust bearings and is operated in a corrosive environment, e.g., the propeller shaft of a marine craft. The various spaces in design may be defined as below:

$$\mathcal{D} = \bigcup_{i=1}^{n_d} \vec{d}_i \tag{4}$$

Example:

$$\vec{d}_1 = \{alloy\ steel, stepped\ shaft, \ldots\} \tag{5}$$

$$\mathcal{M} = \bigcup_{j=1}^{n_m} \vec{m}_j \tag{6}$$

Example:

$$\vec{m}_1 = \{cutting\ speed, feed, \ldots\} \tag{7}$$

$$\mathcal{P} = \bigcup_{d=1}^{n_p} \vec{p}_d \tag{8}$$

Example:

$$\vec{p}_1 = \{torque, corrosion\ resistance, \ldots\} \tag{9}$$

$$\mathcal{C} = \bigcup_{k=1}^{n_e} \vec{c}_k \tag{10}$$

Example:

$$\vec{c}_1 = \{pH\} \tag{11}$$

It is to be noted that the elements in each vector can themselves be sets; e.g., *alloy steel* might indicate a family of alloy steels like $\{stainless\ steel, inconel, \ldots\}$. Further, the evaluation function

Figure 1: Spaces and Functional Set Operators in Design.

The *manufacturing space* $\mathcal{M}$ comprises manufacturing vectors $\vec{m}$, where each vector represents a candidate manufacturing process. The size of this space is determined by the capacity and availability of production facilities. Elements of each vector indicate the various choices for process parameters, including, for example, cutting conditions and process capabilities.

Performance vectors $\vec{p}$ constitute the *performance space* $\mathcal{P}$. These vectors are interpreted as the target performance criteria for a design. They also correspond to an explicit representation of the design or product specifications, typically known as functional requirements.

The *context space* $\mathcal{C}$ contains information about the context or environments in which the design is expected to operate. This is important because life-cycle issues such as environmental factors have an impact on the design vectors. For example, a shaft diameter of a mechanism may require a modification to accommodate temperature changes in the environment. The context space is made up of context or life-cycle vectors $\vec{c}$.

2.2 Functional Set Operators in Design

The design process is represented by the above spaces and several functions that operate on these spaces (refer to Figure 1). The first such function is the fabrication function which maps the idealized design vectors and manufacturing vectors to actual product or part models. This simply means that the design and manufacturing vectors are "combined" to represent variability or deviations as a consequence of manufacturing processes. This operation is symbolically represented as:

$$\Xi[\vec{d}_i, \vec{m}_j] \rightarrow \vec{a}_{ij} \tag{1}$$

where Ξ is the fabrication operator and $\vec{a}_{ij}$ is the actual part model.

An evaluation function is defined to map the actual part model and a context vector $\vec{c}_k$ of the context space to the predicted performance vector $\vec{p}\,^{p}_{ijk}$:

$$\mathcal{H}[\vec{a}_{ij}, \vec{c}_k] \rightarrow \vec{p}\,^{p}_{ijk} \tag{2}$$

In current industrial settings, the development of a complete scientific basis for tolerances is an imperative issue. Other mandatory features that must be incorporated in a comprehensive tolerancing scheme are: compatibility for computer representation and implementation, correspondence between manufacturing errors and performance acceptance indices, inter-relationship between tolerances and measurement parameters, and algorithms to ensure assemblability in mating parts. This research effort is directed towards development of a theory and the methods that will satisfy the above demands. As the primary motivation is to develop tolerance tools for design, a new concept for establishing the tolerance-function connection is introduced in the next section.

1.4 Tolerance and Function: The Total Variability Concept

According to current practices, a component is deemed to be *in spec* if the extremum of the errors lies within a prescribed range or tolerance zone. A tolerance zone is determined by the maximum peak-to-valley difference of a component's surface profile. Such a tolerance zone concept does not convey full information about the component's form error and how it relates to product function [10]. The *total variability concept* is proposed in [37] as an alternative error representation, where fractal-based parameters characterize the total complexity of a manufactured profile. Through this characterization, design performance becomes a direct and explicit function of the fractal parameters. In turn, the performance is directly linked to the potential manufacturing processes, represented, abstractly, by these fractal-based parameters.

The following section presents a general philosophy of tolerance methods in design and manufacturing, motivated by a set-theoretic definition. An overview of previous work on the fractal-based representation of tolerances is presented next. Subsequent sections describe the current work on modeling errors in manufacturing processes, developing a suitable test-bed for fractal-based representations, and experimental results.

2 A Set-Based Approach to Design for Manufacturing

Traditionally, mechanical design implies the design of single objects, i.e., a functional requirement is transformed into a design description representing *one* physical object. In order to develop a more universal model for mechanical design, a set-based approach is proposed by Ward and Seering [35], Wood and Antonsson [36], and Otto [20]. For example, when the design description contains tolerance information, it represents a *set* of alternatives, as opposed to a single part. A set-based approach to design for manufacturing combines the concepts presented in the above references and has the following constituent spaces: design space, manufacturing space, performance space, and context space. These constituent spaces are discussed below, along with set operations, an example to illustrate the fundamental ideas, and the potential role of fractal-based tolerance methods in abstractly linking the design and manufacturing spaces.

2.1 Spaces in the Design Process

The *design space* or product superset $\mathcal{D}$ contains design vectors $\vec{d}$ as its elements; these vectors are the possible solutions to the design problem, evolving as the design is refined. Elements of each vector include design parameters such as material properties, dimensions, and so on. A toleranced dimension in this space implies a subset of all possible dimensions within the tolerance limits.

mechanical tolerances is adopted as the fundamental hypothesis. This paper examines the current issues in tolerancing, indicates the utility of a fractal-based approach, and further describes the current status of dynamic model development, computational representation of tolerances, and experimental work. The following section presents an overview of the information transfer between the design and manufacturing domains and the role of tolerances therein.

1.2 Information flow in DFM

Engineering design is primarily an informational domain, where customer requirements are transformed into a form description and communicated to manufacturing. Manufacturing, a predominantly physical domain, involves the realization of the artifact; in the course of this realization, inherent variabilities in the manufacturing process and material and tool properties result in parts with non-ideal geometries and material characteristics. The need to exercise control on these deviations requires information feedback of various error sources from manufacturing to design. This information is incorporated in subsequent design specifications as tolerances of size and form, surface finish, and so on. The dual role of this variational information, as indicators of process deterioration in manufacturing and as performance delimiters in design, highlights their criticality in DFM. However, the evolution of current tolerancing practices is based on empiricism rather than scientific or mathematical theory. This problem has recently attracted considerable research attention [30]. The next section presents an overview of current tolerancing practices and their limitations.

1.3 Present Tolerancing Methods

Tolerances were formally introduced in American industry in the early 1900's [33]. Considering this late inception, the concept of tolerancing has developed rapidly to its current conventions as embodied in ANSI Y14.5M-1982. A detailed review of current tolerance analysis and synthesis methods is presented in [17]. The following is a brief survey of the more recent tolerance theories.

The *offset boundaries* theory proposed by Requicha [23] prescribes tolerance zones for surface features by generating inner and outer offsets of the nominal features. The theory further suggests a combination of individual feature offset boundaries to form a single offset boundary for the part. This combination creates potential problems in the dependencies between individual feature constraints [32]. In addition there is no explicit correspondence to the manufacturability and function of mechanical features [34]. Another related approach is the theory developed by Srinivasan and Jayaraman [14], where a part geometry is modeled using virtual boundaries, considering all tolerance effects. But if individual feature tolerances are critical to function, this theory does not reflect their impact. Turner and Wozny have introduced the *M-space theory* for tolerances, where each allowable part variation is represented by a model variable, and tolerance specifications are represented as constraints on a vector space of model variables. Turner and Wozny use linear programming methods for tolerance analysis as part of the M-space theory, a potential limitation due to the linearization. Other traditional and modern tolerancing methods are discussed in the authors' earlier publications [37, 29, 28]. Most of the above methods treat the tolerancing problem in a specific, limited context, such as function, manufacturing, or representation. The ubiquity of tolerances in most life-cycle phases is rarely recognized or addressed in tolerance research. This project aims at such a global consideration of tolerances.

Fractal-Based Tolerancing: Theory, Dynamic Process Modeling, Test Bed Development and Experiments[*]

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Abstract

Tolerancing information is a vital ingredient in design for manufacturing. This research aims at developing a global, unified basis for tolerancing, to promote consistent and efficient information flow between the design and manufacturing areas. In this regard, the concepts of fractal-based tolerancing, proposed previously by the authors, are further investigated. Extensions to the study of manufacturing errors, and to the computational representation of fractal-based tolerances, along with the experimental verification of fractal structure in the tolerance range, form the ternary foci of this paper. Some crucial theoretical issues are also identified for further study and development.

1 Introduction

1.1 The Product Life Cycle and Design for Manufacturing

Engineering design has undergone a gradual, yet very prominent metamorphosis over the last decade. In the past, designers concentrated almost exclusively on the functional and performance requirements of the product [19]. The subsequent manufacture, assembly, and inspection were isolated phases with little mutual communication and coordination. However, competition, quality and productivity awareness, and other factors have driven modern industries to adopt a more integrated and coordinated approach to the entire *product life cycle*. This integrated approach results in a broader scope for product design, encompassing manufacturing, assembly, inspection, installation, maintenance, and service. Simultaneous consideration of all the above issues has led to the evolution of special areas of research [12], e.g., concurrent or simultaneous engineering, design for manufacture, design for assembly, design for quality, and others. This project addresses the design for manufacturing (DFM) area, involving the simultaneous consideration of design or product technology and manufacturing technology [12, 8]. More specifically it addresses a critical design issue influenced strongly by manufacturing, i.e., mechanical tolerances. A fractal-based approach to

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