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THEORETICAL FOUNDATIONS FOR TUNING PARAMETER TOLERANCE DESIGN

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ABSTRACT

In this paper a novel technique is presented to solve tolerance design problems. To achieve the desired performance tolerance the technique uses a subtle, but significant, change in the design rather than increasing component precision. This change is the addition of a tuning parameter. Statistical models are used to developed framework for the tuning parameter design method. Also developed is a new dimensionless design parameter which ranks candidate tuning parameters. The step-by-step tuning parameter design method is applied to a heavy duty manual stapler as a clarifying example.

KEYWORDS: Tolerance Design, Tuning Parameter Design, Design Principles, Formal Design Methods, Non-dimensional design parameter

1 Introduction

As system or component tolerances become tighter, manufacturing costs generally increases (Chase and Greenwood, 1987; Abdel-Malek and Asadathorn, 1994; Drake, 1997; Speckhart, 1972). To create high precision, or in some cases sufficient precision systems, engineers often rely on "good" design to avoid the need for precision components. These techniques of "good tolerance" design can be classified into tolerance design principles (McAdams and Wood, 1999). In most cases the application of these principles is informal and based on experience and trial and error. The goal of this paper is to provide a theoretical foundation and methodology for formalizing the application of a

tolerance design principle. The tolerance design principle of interest here is the use of a tuning parameter to provide the desired system performance.

A insightful example of a tuning parameter are the adjustment parameters on the front wheel of an automobile. To drive and steer correctly (straight) requires precise positioning of the front wheels. As suspension, frame, and steering components are manufactured and assembled, the stack up error in camber, castor, and toe-in cause a car to steer poorly. To manufacture an automobile with enough precision that these tolerance stack ups are small enough to allow satisfactory steering is impractical. To allow accurate and straight steering, adjustment parameters for castor, camber, and toe-in are designed into the steering and wheel mounting system that allow the alignment to be tuned.

Though perhaps not always recognized as such, the use of a tuning parameter in tolerance design is a valuable and common design technique. The usefulness and power of a tuning parameter in design in general is developed by Otto and Antonsson in (Otto and Antonsson, 1993). To move the application tolerance tuning parameter design from informal to formal there is a need for a solid theoretical foundation and design method. In this paper, these first steps toward a complete theory of tuning parameter design are presented.

The objective of this method is to develop a formal methodology for tuning parameter design and base it firmly in practical engineering techniques. Such an objective motivates several key questions. How does a designer know when a tuning parameter is needed? What is a useful model of a tuning parameter? What are the performance requirements of the tuning parameter? And

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which of the existing design parameters serves as a better candidate for the addition of tunability? These questions are answered in this paper through a step-by-step method. The steps provide the framework for a tuning parameter design method as well as organization of this paper. A simple example is also presented to clarify the procedure.

2 Step 1: Determining Need

The first step in designing a tuning parameter for tolerance design is to determine if one is needed. There is no need to add additional complexity or manufacturing steps if a product performance is within the desired tolerance. The simple question is, given a performance parameter, a performance parameter tolerance requirement, a design solution, a chosen manufacturing process and the associated manufacturing tolerances, does the system perform within the desired tolerance specification?

Here, the question is answered formally and generally using a statistical model of the variational relationship between the performance and the design parameters.

2.1 An Engineering Model for Tuning Parameter Tolerance Design

Here, a model is developed for tolerance design and analysis. The development and presentation of this model is important for several reasons. One key goal of this model development is the provision of a solid analytical foundation for tuning parameter design. Also, this model development is important to show the integration of tuning parameter design as a general tolerance design tool.

The discussion in this section, and the remainder of this paper, assumes that the performance metric of interest can be represented parametrically. In other words, the product has some performance parameter p . This performance parameter may be a length, a mass, a time, an energy, or any other metric that affects customer needs. This performance parameter can be expressed as a function of design parameters d_i in the form

$$p = f(d_1, d_2, \dots, d_n). \quad (1)$$

An assumption is made here that Equation 1 describes the system completely. The term completely is interpreted in the following manner. A design change occurs in the value of p if, and only if, there is some change in at least one of the design parameters d_i .

It is a common and reasonable assumption in tolerance design and analysis to assume that the design variables in Equation 1 occur as normally distributed random variables (Bowker and Lieberman, 1959; Creveling, 1997; Chase and Greenwood, 1987). In fact, leaders in manufacturing often require suppliers to provide statistical verification of conformance to design specifications (Wright, 1990).

Throughout this paper, the terms *variation*, *tolerance*, *natural tolerance*, *variance*, and *standard deviation* are used. They are used with specific meanings and for clarity of understanding should not be confused with one another. *Variation* means some change or error in some parameter from a ideal or nominal value. *Tolerance* is the specified maximum acceptable variation from the target value. *Natural tolerance* is used to describe the actual variation of a design parameter. In other words, if the tolerance is greater than the natural tolerance, the performance or part is within the design specification.

In the literature (Bowker and Lieberman, 1959), the term natural tolerance more often refers to the variation of a single component feature that results from some specific manufacturing or fabrication techniques. Here the definition of natural tolerance is extended to include the variation of some system as it occurs with some specified configuration and manufacturing process. The term *variance* is used with its common statistical meaning. Variance refers to the second moment about the mean of a randomly occurring value.

2.2 Developing the Mathematical Model

There are two equations that are derived here. The first is the system response of Equation 1 based on the assumptions that each of the design variables d_i are random and normally distributed. The second equation of interest is the standard deviation or variance of p based on these same assumptions. In each of these derivations, it is assumed that the design variables are independent. The expected value of the system performance is determined first.

2.2.1 Deriving the System Response Restating Equation 1,

$$p = f(d_1, d_2, \dots, d_n). \quad (2)$$

The derivation proceeds by taking a Taylor series expansion of f about the expected values of the design values: $[E(d_1), E(d_2), \dots, E(d_n)]$. The expansion gives

$$\begin{aligned} f(d_1, d_2, \dots, d_n) &\approx f[E(d_1), E(d_2), \dots, E(d_n)] \\ &+ \sum_{i=1}^n \frac{\partial f}{\partial d_i} [d_i - E(d_i)] \\ &+ \frac{1}{2} \left\{ \sum_{i,j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j} [x_i - E(x_i)] [x_j - E(x_j)] \right\} \end{aligned} \quad (3)$$

discarding terms third order and higher. In Equation 3 all the derivatives are evaluated at the expected values. In other words

$$\frac{\partial f}{\partial d_i} = \frac{\partial f(d_1, d_2, \dots, d_n)}{\partial d_i} \Bigg|_{d_l = E(d_l), l=1,2,\dots,n} \quad (4)$$

Taking the expected value of both sides of Equation 3 gives

$$E[f(d_1, d_2, \dots, d_n)] = E\{f[E(d_1), E(d_2), \dots, E(d_n)]\} \\ + E\left\{\sum_{i=1}^n \frac{\partial f}{\partial d_i} [d_i - E(d_i)]\right\} \\ + E\left\{\frac{1}{2} \sum_{i,j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j} [x_i - E(x_i)][x_j - E(x_j)]\right\}. \quad (5)$$

Recalling the properties of the expected value operator when applied to independent random variables

$$E\{[x_i - E(x_i)]^r [x_j - E(x_j)]^s\} = E[x_i - E(x_i)]^r E[x_j - E(x_j)]^s, i \neq j, \quad (6)$$

$$E(x_1 + x_2 + \dots + x_n) = E(x_1) + E(x_2) + \dots + E(x_n), \quad (7)$$

$$E(c) = c, \quad (8)$$

with c a constant, and

$$E(cx) = cE(x) \quad (9)$$

were x is any random variable (Hahn and Shapiro, 1994). Applying these to the first term on the right in Equation 5 gives

$$E\{f[E(d_1), E(d_2), \dots, E(d_n)]\} = f[E(d_1), E(d_2), \dots, E(d_n)]. \quad (10)$$

Applying the property given by Equation 8 to the second term in Equation 5 gives

$$E\left\{\sum_{i=1}^n \frac{\partial f}{\partial d_i} [d_i - E(d_i)]\right\} = \sum_{i=1}^n E\left\{\frac{\partial f}{\partial d_i} [d_i - E(d_i)]\right\} \\ = \sum_{i=1}^n \frac{\partial f}{\partial d_i} \{E[d_i - E(d_i)]\} \\ = 0. \quad (11)$$

Applying the property given by Equation 6 to the third term in Equation 5 and using the definition of the variance gives

$$E\left\{\frac{1}{2} \sum_{i,j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j} [x_i - E(x_i)][x_j - E(x_j)]\right\} = \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2} \text{Var}(x_i) \\ + 0. \quad (12)$$

Substituting Equations 10, 11, and 12 back into Equation 5 gives the expected value of the system performance as

$$E(p) = E[f(d_1, d_2, \dots, d_n)] \approx f[E(d_1), E(d_2), \dots, E(d_n)] \\ + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2} \text{Var}(x_i). \quad (13)$$

Though not a true equality (it is an approximation), from here forward, the system response will be written as

$$\bar{p} = f(\bar{d}_1, \bar{d}_2, \dots, \bar{d}_n) + \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2} \text{Var}(x_i), \quad (14)$$

where

$$\bar{p} = E(p) \quad (15)$$

and

$$\bar{d}_i = E(d_i). \quad (16)$$

2.2.2 Deriving the System Variance Here, the variance on the system performance is determined. The derivation retains terms up to the second order.

The variance of p is

$$\text{Var}(p) = E(p^2) - [E(p)]^2 \quad (17)$$

from the definition of the variance operator (Bowker and Lieberman, 1959). Substituting Equation 1 into Equation 17 gives

$$\text{Var}[f(d_1, d_2, \dots, d_n)] = E[f(d_1, d_2, \dots, d_n)]^2 - \{E[f(d_1, d_2, \dots, d_n)]\}^2 \quad (18)$$

To obtain an approximation to the first term on the right-hand side of Equation 18, the square of Equation 3 is taken and expected values are taken on a term by term basis (employing Equation 7). Terms are retained up to second order. This approach leads to

$$E\{[f(d_1, d_2, \dots, d_n)]^2\} = \{f[E(d_1), E(d_2), \dots, E(d_n)]\}^2 \\ + \sum_{i=1}^n \left(\frac{\partial f}{\partial d_i}\right)^2 E[d_i - E(d_i)]^2 \quad (19)$$

the terms not shown here are all higher than second order. The second term on the right hand side of Equation 18 is the square of Equation 13 thus

$$\{E[(f(d_1, d_2, \dots, d_n)]\}^2 = \{f[E(d_1), E(d_2), \dots, E(d_n)]\}^2. \quad (20)$$

Substituting Equations 19 and 20 into Equation 18 gives the equation for the system variance:

$$\text{Var}[f(d_1, d_2, \dots, d_n)] = \sum_{i=1}^n \left(\frac{\partial f}{\partial d_i}\right)^2 E[d_i - E(d_i)]^2. \quad (21)$$

Using the definition of the variance on the design variables gives,

$$\text{Var}(p) = \text{Var}[f(d_1, d_2, \dots, d_n)] = \sum_{i=1}^n \left(\frac{\partial f}{\partial d_i} \right)^2 \text{Var}(d_i). \quad (22)$$

It is more common to describe tolerances in terms of the standard deviation rather than the variance. To express Equation 22 in terms of the standard deviation, the square root of both sides gives

$$\sigma_p = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial d_i} \right)^2 \sigma_i^2} \quad (23)$$

where σ_i is the standard deviation of the design parameter d_i .

2.2.3 Determining Need It is common practice in engineering to interpret the tolerance specification as $N\sigma$ (Creveling, 1997). For example, if the target value and tolerance for the weight of a box of breakfast cereal is listed as $24 \pm 1oz$, the standard deviation of the weight of the box is $1/Noz$. The analogous relation holds between the natural tolerance and standard deviation. Generally, N is taken to be 3. For $N = 3$ (Creveling, 1997), the probability of the performance being within $p \pm 3\sigma_p$ is 99.7% (Hahn and Shapiro, 1994). This convention is used for the remainder of this paper. In other words, the term tolerance implies a system performance requirement on the standard deviation of $\sigma_p = \text{tol}/N$ with $N = 3$. Similarly, the natural tolerance of the system is interpreted as $\text{ntol} = N\sigma_p = 3\sigma_p$.

Using this convention and the equations developed above, it is a straight forward calculation to determine if a tuning parameter is needed. Using Equation 23, if the condition

$$\text{tol}/3 < \text{ntol}/3 = \sigma_p = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial d_i} \right)^2 \sigma_i^2} \quad (24)$$

is violated, a tuning parameter is needed; i.e. if the natural tolerance exceeds the tolerance, the addition of tunability is required.

3 Step 2: Performance Requirements for Tuning Parameter Candidates

The next step in the tuning parameter design process is to determine the performance requirements for candidate tuning parameters. The goal of the method presented in this paper is to use an engineered, rather than intuitive, approach to determine which design parameters are superior candidates for the addition of tunability. To achieve this goal, the tuning parameter performance requirements for multiple design parameters are found and compared in Step 3 to determine which design parameters are more

appropriate candidates for the addition of tunability. Also, this performance requirements for a tuning parameter are used as design constraints in Step 4 as tunability is designed into the system. Because of their use in the next steps, the performance requirements are discussed and developed as Step 2.

There are two key performance requirements for a tuning parameter. The first critical requirement of this parameter is that, through some range in its adjustment, an out of specification performance metric be tuned to the target value. The second important performance parameter is that the tuning parameter provide the required performance tolerance for the system. First the range requirement is discussed. The tolerance requirement for the tuning parameter is discussed next.

3.1 Determining the Tuning Parameter Range Requirements

Figure 1 shows the performance parameter distribution that is implied by some performance specification $p \pm \text{tol}$. Also shown, as the dotted line, is the performance parameter distribution of the actual design. As shown, the natural tolerance is larger than the specified tolerance. Thus, as discussed in the previous section, a tuning parameter needs to be added to the system.

Consider a single product taken from the distribution, with performance p^* as shown in Figure 1. The requirement of the tuning parameter is that when this value is some $p^* \neq \bar{p}$ then the value of d_k is changed to some new value d_k^t such that $p^* = \bar{p}$. Here, the parameter to which tunability is added is denoted d_k . This condition can be stated more explicitly as follows. If the particular value of a performance metric, p , is

$$p^* = f(d_1^*, d_2^*, \dots, d_k, \dots, d_n^*) \neq \bar{p} \quad (25)$$

where the superscript * indicates the specific values that the design parameters have taken to cause p^* . Then d_k must be changed to d_k^t such that

$$p^t = f(d_1^*, d_2^*, \dots, d_k^t, \dots, d_n^*) = \bar{p} \quad (26)$$

where p^t is the value achieved by tuning the system. The different values to which d_k^t must adjusted is a key performance requirement for the tuning parameter. Inverting Equation 26 and solving for the tuning parameter gives

$$d_k^t = f^{-1}(\bar{p}, d_1^*, d_2^*, \dots, d_{k-1}^*, d_{k+1}^*, \dots, d_n^*) \quad (27)$$

Thus, the particular value of d_k^t needed to tune can be determined from the original model.

To determine the adjustability requirement on d_k , a statistical worst case condition is considered for the design variables $d_i \forall i \neq$

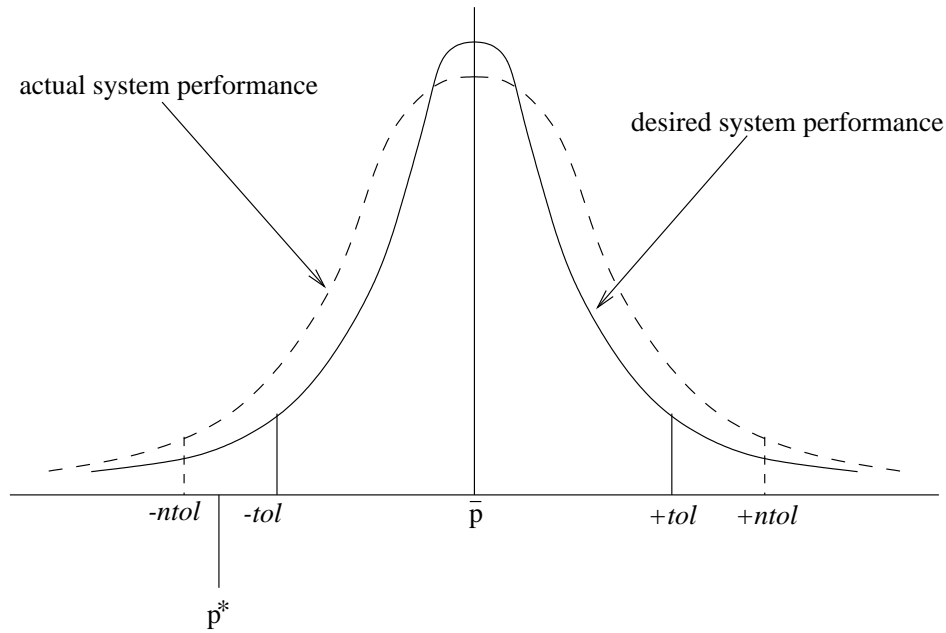


Figure 1. The response of a performance parameter. The solid line represents the desired response. The dotted line represents the actual response.

k . The “worst” possible value that occurs for each is $d_i^{max} = \bar{d}_i + ntol_{d_i}$ or $d_i^{min} = \bar{d}_i - ntol_{d_i}$. The adjustable range requirement for d_k is expressed as the solution to two constrained optimization problems:

$$d_k^{low} = \min[f^{-1}(\bar{p}, d_1, d_2, \dots, d_{k-1}, d_{k+1}, \dots, d_n)] \quad (28)$$

subject to

$$d_i^{min} \leq d_i \leq d_i^{max}, \quad i = 1, 2, \dots, k-1, k+1, \dots, n, \quad (29)$$

and

$$d_k^{high} = \max[f^{-1}(\bar{p}, d_1, d_2, \dots, d_{k-1}, d_{k+1}, \dots, d_n)] \quad (30)$$

subject to

$$d_i^{min} \leq d_i \leq d_i^{max}, \quad i = 1, 2, \dots, k-1, k+1, \dots, n. \quad (31)$$

Simply put, the requirement on d_k is that it have some range of adjustability from d_k^{low} to d_k^{high} such that it can always tune p to \bar{p} for any value the d_i that occurs. Though the solution to $2 \times n$ optimization problems is potentially computationally time consuming, the worst case range on the design variables d_i is generally “small” thus reducing the optimal search space on the constraint requirement. In practice, due to this “small” search space, the extrema very often lie on the search space boundaries.

3.2 Tuning Parameter Tolerance Requirements

The addition of a tuning parameter essentially removes the random effects of the design parameters that are not tuned. The value of d_k^t is set in *response* to the values the design parameters have taken for single case. Thus, from the perspective of the performance, the untuned design parameters are no longer stochastic variables but fixed constants. From the perspective of the output performance, however, the tuning parameter remains a stochastic variable. The addition of tuning parameter adds some new set of design parameters to the system. Also, there is some tuning procedure, such as aligning the front the steering system discussed in Section 1, that results in a random variation of the performance. Using this model, each particular value of d_k^t is actually some expected value, $E(d_k^t)$, with some standard deviation $\sigma_{d_k^t}$.

Proceeding with a derivation similar to that presented in Step 1, the standard deviation of the system can be represented as

$$\sigma_p = \left| \left(\frac{\partial f}{\partial d_k} \right) \right| \sigma_{d_k} \quad (32)$$

because there is only one random variable now, d_k . Using Equation 32, the tolerance requirement for the tuning parameter is expressed in terms of the system tolerance requirement as

$$tol_{d_k} = 3\sigma_{d_k} = \frac{1}{\left| \left(\frac{\partial f}{\partial d_k} \right) \right|} 3\sigma_p = \frac{1}{\left| \left(\frac{\partial f}{\partial d_k} \right) \right|} tol_p. \quad (33)$$

By substituting the system tolerance requirement into Equation 33, the tolerance requirement for each candidate is determined.

3.3 Summarizing the Results

Each design parameter, if used as the tuning parameter, has different individual performance requirements. These parameters are used in the next step to determine which is the most suited for the addition of tunability. Also, one the desired tuning parameter is chosen, it must be designed to meet these performance requirements. Table 1 is used to organize the performance requirements so they are easily referenced during the tuning parameter design process.

parameter	d_k^{low}	d_k^{high}	tol
d_1	d_1^{low}	d_1^{high}	tol_1
d_2	d_2^{low}	d_2^{high}	tol_2
.	.	.	.
.	.	.	.
.	.	.	.
d_n	d_n^{low}	d_n^{high}	tol_n

Table 1. A general example of the results of the step 2 of the tuning parameter design.

Shown in Table 1 is the results of step 2 for a general case. Table 1 is filled out by considering each design parameter as a tuning parameter and calculating the performance requirements.

4 Step 3: Selecting the Preferred Tuning Parameter

The next step in the process is to select a most appropriate tuning parameter. The “ideal” tuning parameter requires a small range of adjustability to control the output while at the same time the stochastic variability in the tuning parameter cause little variability in the output. It is on these merits that the different candidates tuning parameters are compared.

The first measure of tuning parameter superiority is the range that it requires to control the system as discussed in step 3. The range is defined as

$$R_k = d_k^{high} - d_k^{low}. \quad (34)$$

The smaller this value, the less the design parameter d_k , and thus the design, has to be adjusted and altered. The second measure

of tuning parameter superiority is it’s sensitivity coefficient;

$$S_k = \left| \frac{\partial f}{\partial d_k} \right|. \quad (35)$$

Simply put, the smaller S the more error allowed in setting d_k^t during adjustment.

To simplify the comparison between different tuning parameters, these two measure can be combined into one. For both of these these measures, smaller is better, so they can be multiplied or added to give a single measure. The product of S and R are in the units of p ; thus, multiplying gives superior clarity for comparing a single measure between the tuning parameters . The result of dividing the system performance tolerance by this product yield a nondimensional number that allows all the candidate tuning parameters to be compared. The equation for this measure is

$$\tau_k = \frac{tol_p}{S_k R_k}. \quad (36)$$

Table 2 shows a abstract example using Equations 34, 35, and 36 to rank candidate tuning parameters. In Table 2 the design parameter that is reviewed first for the addition of tunability is d_p . Based on the mathematical behavior of the system, this parameter will be the most advantageous to tune.

d	S	R	τ	rank
d_1	S_1	R_1	τ_1	5
d_2	S_2	R_2	τ_2	3
.
.
.
d_p	S_p	R_p	τ_p	1
.
.
.
d_n	S_n	R_n	τ_n	2

Table 2. Calculating and ranking the candidate tuning parameters.

5 Step 4: Implementing the Tuning Parameter Concepts

The last step in tuning parameter design is the generation of a concept that changes some design parameter from a value of \bar{d}_k to one that can be adjusted from d_k^{low} to d_k^{high} to meet performance requirements discussed in step 2. The solution chosen depends on the constraints and physics of the system and the tuning parameter.

For example, consider the performance requirement of smooth rotation, or balance, for a rotating system. The tuning parameter chosen is the location of a center of mass. The tuning parameter design solution may be either to remove or add mass at a specific point. Whether or not one of these solutions is superior depends on the physics and constraints of the design that may not be represented in Equation 1. For example, when balancing the wheels on an automobile, mass is added using small lead weights. In this case, adding mass is the superior solution due to the clear negatives of removing mass from the rim or tire. On the other hand, when balancing electric motor rotors in hand held power drills, mass is generally removed (by drilling or grinding away part of the rotor core) because spatial constraints make the addition of material problematic.

6 Summary - Tuning Parameter Design Method

This section is a short summary of the tuning parameter design. This summary may be used as a method and guide when performing tuning parameter design.

1. Determine Need

The performance model

$$p = f(d_1, d_2, \dots, d_n),$$

is developed. The nominal values for p and d_i are developed based on customer needs. The natural tolerances for the design parameters are determined based on manufacturing process and assembly models. System tolerance requirements and design parameter natural tolerances are substituted into

$$tol/3 < ntol/3 = \sigma_p = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial d_i}\right)^2 \sigma_i^2},$$

to determine if a tuning parameter is needed. If a tuning parameter is not needed, then the tolerance design problem is solved.

2. Determine Tuning Parameter Performance Requirements

The tuning requirements d_k^{low} and d_k^{high} for each design parameter are determined using

$$d_k^{low} = \min[f^{-1}(\bar{p}, d_1, d_2, \dots, d_{k-1}, d_{k+1}, \dots, d_n)]$$

subject to

$$d_i^{min} \leq d_i \leq d_i^{max}, \quad i = 1, 2, \dots, k-1, k+1, \dots, n,$$

and

$$d_k^{high} = \max[f^{-1}(\bar{p}, d_1, d_2, \dots, d_{k-1}, d_{k+1}, \dots, d_n)]$$

subject to

$$d_i^{min} \leq d_i \leq d_i^{max}, \quad i = 1, 2, \dots, k-1, k+1, \dots, n.$$

The tolerance requirements tol_k are determined for each design parameter using

$$tol_{d_k} = 3\sigma_{d_k} = \frac{1}{\left|\left(\frac{\partial f}{\partial d_k}\right)\right|} 3\sigma_p = \frac{1}{\left|\left(\frac{\partial f}{\partial d_k}\right)\right|} tol_p.$$

These performance requirements are then tabulated as in Table 1 for future reference in steps 3 and 4.

3. Rank the tuning parameters

The range requirements for each candidate tuning parameter is determined using

$$R_k = d_k^{high} - d_k^{low}.$$

The sensitivity coefficients are determined using

$$S_k = \left| \frac{\partial f}{\partial d_k} \right|.$$

The tuning parameter suitability measure is determined using

$$\tau_k = \frac{tol_p}{S_k R_k}.$$

The data are organized as in Table 2. The candidate tuning parameters are ranked according to their τ value, higher being better.

4. Generate concepts that satisfy the design requirements from Step 2 to implement the tuning parameter.

The equations, concepts, and requirements for tuning parameter design are now developed generally. Also, they are presented in a step-by-step method. This step-by-step method is now applied specifically to a simple design problem.

7 A Clarifying Design Example

In this section, a simple design example is presented to clarify the concepts and show the implementation of the methods presented in the above sections. The example is the tolerance design of a spring that provides the function of storing energy. The spring comes from a heavy duty construction stapler as shown in Figure 2.

Table 3 contains design and performance information for the spring system parameters. As a result of minimum energy requirements to sink a staple and maximum energy requirements to limit user fatigue, the performance requirement for the energy storage is $E = 1.8 \pm 0.05 J$. The variations listed in Table 3 for the spring constant, initial, and final spring lengths are natural tolerances. The design task is the selection of the most preferred one of these design parameters to which tunability is added thus allowing the tolerance to be achieved. To complete the tuning parameter design, a concept is developed to add this tunability.

7.1 Step 1: Determining Need

The key performance metric for the spring is the energy it stores. Thus, the system performance model for the spring is

$$E = \frac{1}{2}K(x_f^2 - x_i^2). \quad (37)$$

Using Equation 23 and the values from Table 3 gives the performance as

$$E = 1.8 \pm 0.10. \quad (38)$$

where 0.10 is the natural tolerance and is defined as $3\sigma_E$. Comparing the natural tolerance of the spring system to the tolerance requirement in Table 3 shows that $tol = 0.05 \neq 0.10 = ntol$; thus, the system needs to be tuned.

8 Step 2: Performance Requirements for Tuning Parameter Candidates

Continuing with the design process, the performance requirements for the candidate tuning parameters are determined.

Beginning the the design parameter of the spring constant, K , the range requirements and tolerance are determined. Solving Equation 37 for K gives

$$K = \frac{2E}{x_f^2 - x_i^2}. \quad (39)$$

The optimization problem, as presented in Equations 28, 29, 30, and 31 is formed for this Equation 39.

$$K^{low} = \min \left[\frac{2\bar{E}}{(x_f)^2 - (x_i)^2} \right], \quad (40)$$

and

$$K^{high} = \max \left[\frac{2\bar{E}}{(x_f)^2 - (x_i)^2} \right], \quad (41)$$

constrained to

$$\begin{aligned} x_f - ntol_{x_f} &\leq x_f \leq x_f + ntol_{x_f} \\ x_i - ntol_{x_i} &\leq x_i \leq x_i + ntol_{x_i}. \end{aligned} \quad (42)$$

In this constrained space, Equation 39 has no extrema, thus k^{high} and k^{low} are found on the borders of the constraint space from Equation 42. To solve the for the lower tunability requirement on K , Equation 39 is written as

$$K^{low} = \frac{2\bar{E}}{(x_f + ntol_{x_f})^2 - (x_i - ntol_{x_i})^2}. \quad (43)$$

Equation 43 is the lower constraint equation, Substituting the values in from Table 3 gives

$$K^{low} = \frac{2 \times 1.8}{(2.0 + 0.02)^2 - (1.0 - 0.02)^2} \frac{J}{cm^2} = 11,538 \text{ N/m}. \quad (44)$$

Similarly, to solve for the K^{high}

$$K^{high} = \frac{2\bar{E}}{(x_f - ntol_{x_f})^2 - (x_i + ntol_{x_i})^2}. \quad (45)$$

Substituting the values in from Table 3 gives

$$K^{high} = \frac{2 \times 1.8}{(2.0 - 0.02)^2 - (1.0 + 0.02)^2} \frac{J}{cm^2} = 12,500 \text{ N/m}. \quad (46)$$

parameter	description	value
E	energy stored in spring	$1.8 \pm 0.05J$
K	spring constant	$12,000 \pm 600N/m$ (SAE, 1996)
x_f	final compression of spring	$2 \pm 0.02cm$
x_i	initial compression of spring	$1 \pm 0.02cm$

Table 3. Tolerance design data for the spring.

To solve for the tolerance, Equation 33 is used. For the spring performance model in Equation 37 this gives

$$tol_K = 3\sigma_K = 3 \frac{1}{\left| \frac{\partial E}{\partial K} \right|} \sigma_E = 3 \frac{1}{\frac{1}{2}(x_f^2 - x_i^2)} \sigma_E \quad (47)$$

where K is the candidate tuning parameter, d_k . Substituting in the appropriate values from Table 3 gives

$$tol_K = 333 \text{ N/m} \quad (48)$$

Repeating this procedure for each of the design parameters, the performance requirements for the other candidate tuning parameters are determined and listed in Table 4.

parameter	d_k^{low}	d_k^{high}	tol
K	$11,538 \text{ N/m}$	$12,500 \text{ N/m}$	333 N/m
x_i	0.87 cm	1.11 cm	0.04 cm
x_f	2.05 cm	1.95 cm	0.02 cm

Table 4. Candidate tuning parameters performance requirements for the spring design.

9 Step 3 - Selecting the Appropriate Tuning Parameter

To rank the tuning parameters, the tuning range requirement, the tuning parameter sensitivity, and the tunability ratio τ are calculated for each of design parameters.

Beginning again with the spring constant, the range R_K is

$$R_K = K^{high} - K^{low} = 12,500 - 11,538 = 961.5 \text{ N/m}. \quad (49)$$

Solving for the sensitivity, S_K and evaluating at the nominal values gives

$$S_K = \left| \frac{\partial E}{\partial K} \right| = \frac{1}{2}(x_f^2 - x_i^2) = 1.5 \times 10^{-4} \text{ m}^2 \quad (50)$$

Writing Equation 36 for the spring gives

$$\tau_K = \frac{tol_E}{S_K R_K} = \frac{(0.05)}{(1.5 \times 10^{-4})(961.5)} = 0.35 \quad (51)$$

Proceeding in a similar fashion, the rest of the data is determined and recorded in Table 2.

d	S	R	τ	rank
K	$1.5 \times 10^{-4} m^2$	961.5 N/m^2	0.35	1
x_i	$120N$	0.24 cm	0.16	3
x_f	$240N$	0.10 cm	0.20	2

Table 5. Calculating and ranking the candidate tuning parameters for the spring design.

Reviewing the tuning parameter ranks in Table 5, shows the spring stiffness as superior tuning parameter candidate. Because of the difficulty of changing the stiffness of a compression coil spring after it has been manufactured however, the spring is not considered further as a tuning parameter candidate.

The next most desirable candidate for a tuning parameter is the final length of the spring. The performance requirements for this design parameter are that it be adjustable from $x_f^{low} = 1.95 \text{ cm}$ to $x_f^{high} = 2.05 \text{ cm}$ and the embodied tuning solution has a tolerance of $\pm 0.02 \text{ cm}$. A simple concept for adding tunability to x_f is presented next.

10 Step 4: Implementing the Tuning Parameter Concepts

The final length of the spring is controlled by a combination of stop tabs on the stapler housing and tabs on the stapler hammer assembly, as Shown in Figure 2 and Figure 3. The positions of these tabs in the sheet metal stamps are such that at x_f , the assembly can move no further. The six bar linkage, shown in Figure 2 slides off the lip (shown in Figure 3) and releases the energy stored in the spring thus providing the transformation of the energy stored in the spring to kinetic energy in the hammer assembly. The staple hammer pushes the staple into the object to be stapled.

A simple solution to tuning the design parameter x_f is grind off the top edge of the stapler hammer as needed. This region is shown in Figure 6. This particular solution for the addition of tunability only allows the design parameter x_f to be shortened. To ensure that the x_f^{high} can be achieved, the nominal length of the stapler hammer must be changed to x_f^{high} . This solution is shown in Figure 6. The critical length in setting x_f is the distance between the locating hole and the top of the stop tab. The overall length of the stop tab is increased to from 1.5 cm to 1.55 cm. The natural tolerance of the grinding operation used to shorten this length is $\pm 0.005\text{cm}$ (Ryffel, 1988). This tolerance translates directly to the design parameter x_f .

Using the solution, tunability is added to the store energy. This addition of tuning allows the store energy function of the stapler operate within the natural tolerances of the manufacturing process for both the original design parameters, K and x_i as well as the implementation of the solution for tunability chosen for x_f .

11 Future Work and Conclusions

Though the foundations for tuning parameter design presented here make a significant contribution to tolerance design, some of the assumptions constrain the tuning parameter design problem. The methods presented here are for designing tunability, or adding functionality, into an existing design parameter. In the above development the assumption is made about the completeness of the design model in Equation 13. Because of this assumption, the addition of “new” tuning parameters is neglected in the sense that “new” means it changes the character or form of Equation 13. For tuning parameter additions that change the parametric representation or perhaps the entire physics of the system, further formal methodologies similar to those presented in this paper are still required.

The approach presented in this paper uses Equation 13 for much of the design method development. For some systems, developing these equations is difficult to the degree that experimental analysis is preferred for performance analysis. If experimentation is needed, the approach to tuning parameter design pre-

sented here remains valid but requires extension. The key extension required is that the sensitives and tuning parameter ranges must be determined experimentally instead of analytically.

This paper develops a formalized, mathematically based method for tuning parameter design. The key contributions of this paper are the development of a foundation for a complete theory for tuning parameter design. Using the methods and tools developed here, tuning parameter design can be performed in a repeatable, constructive manner without under dependence on arbitrary and *ad hoc* design decisions. This method develops quantitative requirements for candidate tuning parameters, thus allowing designers clear insight into answering the key design question of which design parameter should be tuned.

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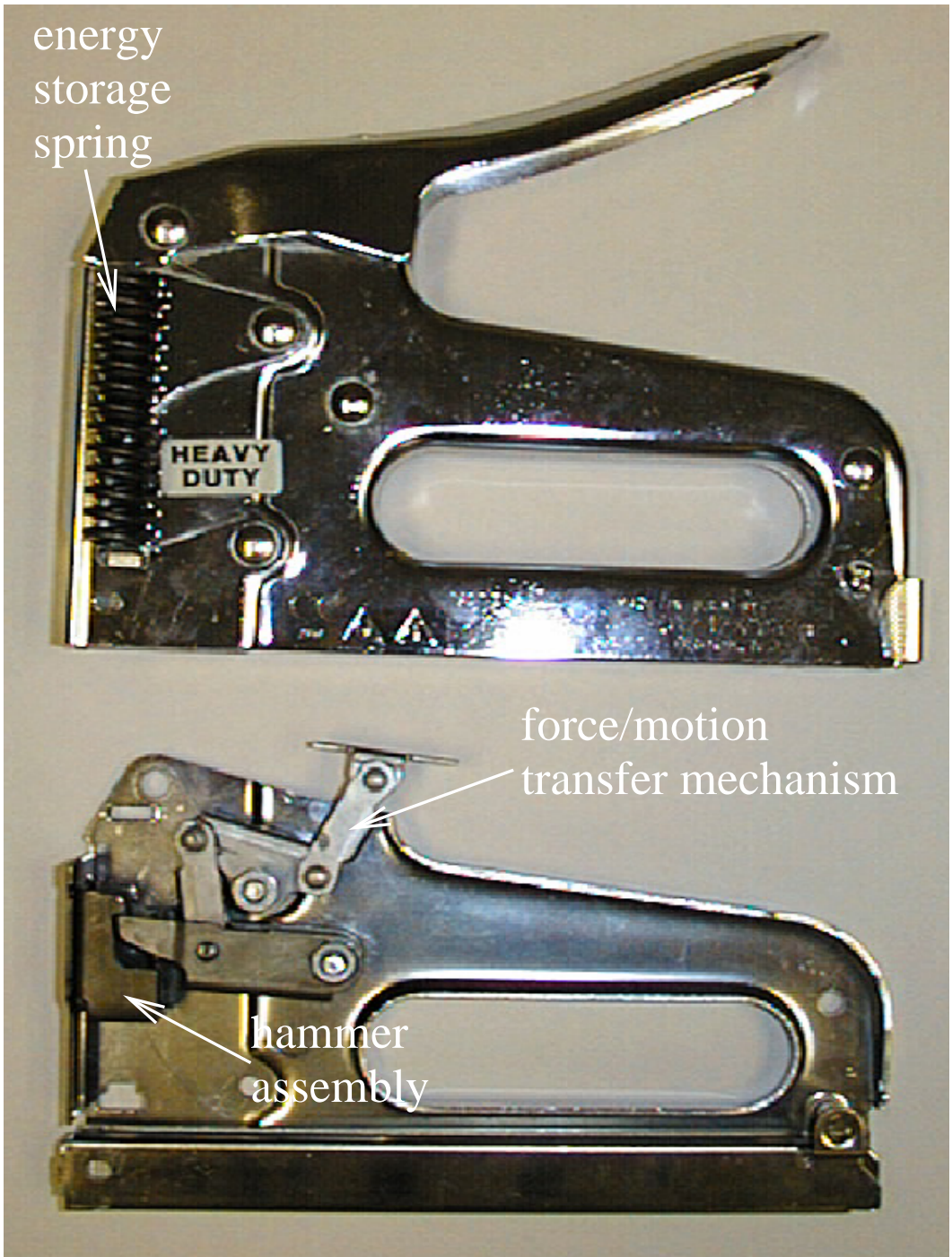


Figure 2. The heavy duty construction stapler.

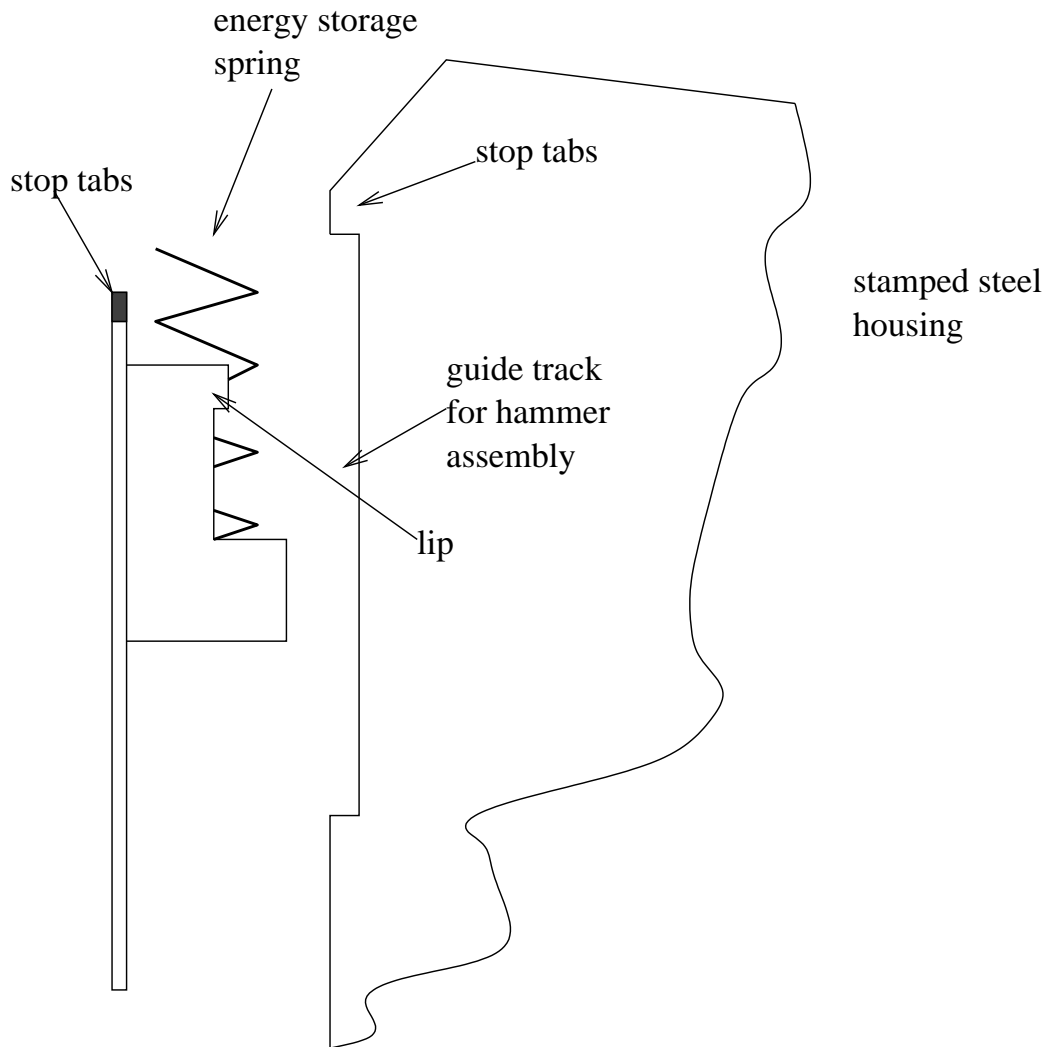


Figure 3. The solution used to control the final length of the spring in the stapler.

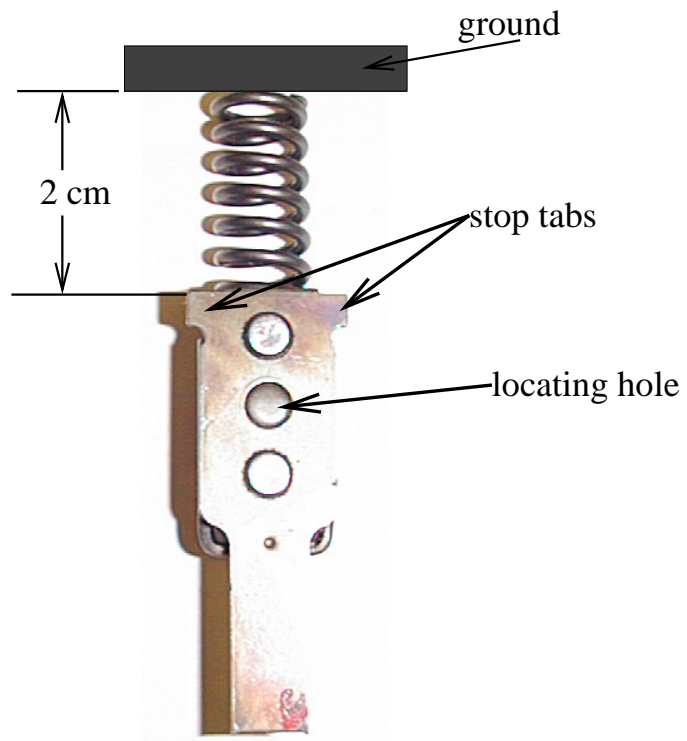


Figure 4. A photo of the hammer assembly with spring.

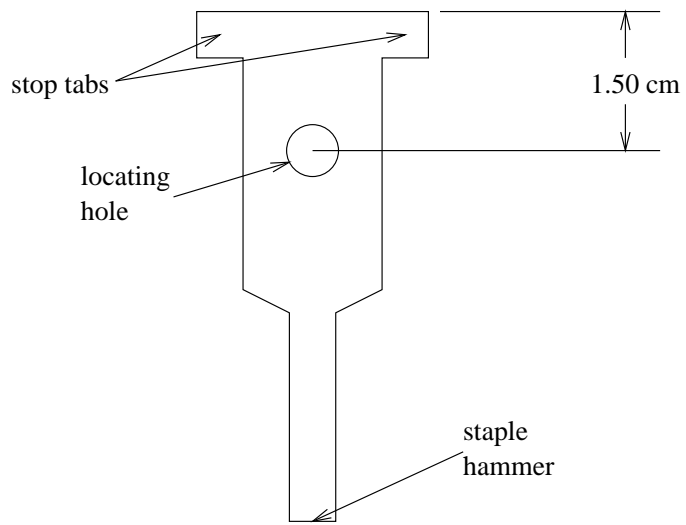


Figure 5. The original hammer plate without the necessary changes for tunability.

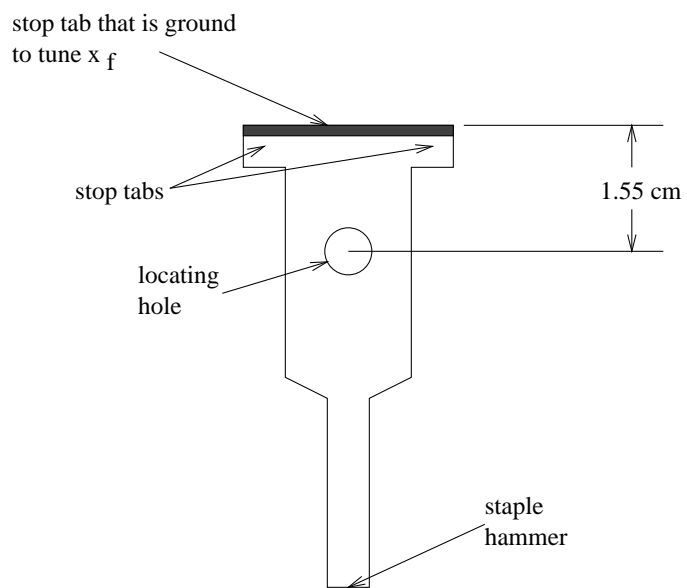


Figure 6. The hammer plate with design changes to allow for tuning