Deadline Differentiated Pricing of Deferrable Electric Loads

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***Abstract*—A large fraction of the total electric load is comprised of end-use devices whose demand for energy is inherently deferrable in time. Of interest is the potential to leverage on such latent flexibility in demand to absorb variability in power supplied from intermittent renewable generation. A fundamental challenge lies in the design of incentives that induce the desired response in demand. With an eye to electric vehicle charging, we propose a novel forward market for deadline-differentiated electric power service, where consumers consent to deferred service of pre-specified loads in exchange for a reduced per-unit price for energy. The longer a consumer is willing to defer, the lower the price for energy. The proposed forward contract provides a guarantee on the aggregate quantity of energy to be delivered by a consumer-specified deadline. Under the earliest-deadline-first (EDF) scheduling policy, which is shown to be optimal for the supplier, we explicitly characterize a non-discriminatory, deadline-differentiated pricing scheme that yields an efficient competitive equilibrium between the supplier and consumers. We further show that this efficient pricing scheme, in combination with EDF scheduling, is incentive compatible (IC) in that every consumer would like to reveal her true deadline to the supplier, regardless of the actions taken by other consumers.**

***Index Terms*—Demand response, electricity markets, incentive compatibility**

I. Introduction

As the electric power industry transitions to a greater reliance on intermittent and distributed energy resources, there is an increasing need for flexible resources that can respond dynamically to weather impacts on wind and solar photovoltaic output. These renewable generation sources have limited controllability and production patterns that are intermittent and uncertain. Such variability represents one the most important obstacles to the deep integration of renewable generation into the electricity grid. The current approach to renewable energy integration is to balance variability with dispatchable generation. This works at today's modest penetration levels, but it cannot scale, because of the projected increase in reserve generation required to balance the variability in renewable supply [7]. If these increases are met with combustion fired generation, they will both be counterproductive to carbon emissions reductions and economically untenable.

As wind and solar energy penetration increases, how must the assimilation of this variable power evolve, so as to minimize these integration costs, while maximizing the net environmental benefit? Clearly, strategies which attenuate the increase in conventional reserve requirements will be an essential means to this end. One option is to harness the flexibility in consumption on the demand side. As such, significant benefits have been identified by the Federal Energy Regulatory Commission [12] in unlocking the value in coordination of demand-side resources to address the growing need for firm, responsive resources to provide supply-demand balancing services (ancillary services) for the bulk power system.

A. The Current Approach to Demand Response

There is an opportunity to transform the current operational paradigm, in which supply is tailored to follow demand, to one in which *demand is capable of reacting to variability in supply* – an approach which is generally referred to as demand response (DR) [1]. The primary challenge is the *reliable extraction of the desired response* from participating demand resources on time scales aligned with traditional bulk power balancing services.

The majority of DR programs in place today are limited to peak shaving and contingency-based applications. The two most common paradigms for customer recruitment and control are: (1) *direct load control* where a load aggregator or utility procures the capability of direct load adjustment through a forward transaction (e.g. call options for interruptible load) and (2) *indirect load control* where consumers themselves adjust their energy consumption in response to dynamic (time-varying) prices [13]. While dynamic pricing has the potential to improve the economic efficiency of electricity markets [5], [29], [14], it subjects consumers to the risk of paying high peak prices, and can have the counterproductive effect of increasing the variability in demand [16], [25]. And, of particular relevance to this paper's emphasis on electric vehicle (EV) charging is a recent empirical study [27] that indicates dynamic pricing may perform worse than a flat-rate tariff for EV charging in terms of generation costs and emissions impacts. In short, demand response implemented through dynamic pricing may not provide the level of assurance required to avoid the use of conventional generation to manage the electric power system.

B. A Deadline Differentiated Energy Service Approach

In the following paper, we propose a novel market framework to enable the direct control of deferrable loads, with a particular emphasis on electric vehicle (EV) charging. Broadly, the proposed market centers on the provisioning of deadline differentiated energy services to customers possessing the inherent ability to delay their consumption up to a point) in time.

From the consumer's perspective, the longer she is willing to delay the receipt of a specified quantity of energy, the less

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that customer pays (per-unit) for said energy. The supplier, on the other hand, implicitly purchases the right to manage the real-time delivery of power to participating consumers by offering a discount on energy with longer deadlines on delivery. And, the longer the consumer-specified deadlines, the more flexibility the supplier has in meeting the corresponding energy requirements. Put simply, the supplier can extract flexibility from the demand-side through direct load control, all while providing firm guarantees on delivery by consumer-specified deadlines. In this way, the supplier can align its operational requirements with the vast heterogeneity in end-use customer needs.

The general concept of service differentiation is not new [21], [20]. Many have studied the problem of centralized control of a collection of loads for load-following or regulation services – all while ensuring the satisfaction of a pre-specified quality-of-service (QoS) to individual resources [6], [9], [17], [18], [22], [24], [32], [35] There has, however, been little work in the way of designing market mechanisms that endogenously price the flexibility being offered by the demand side, while incentivizing consumers to truthfully reveal their preferences to the operator – for example, the ability to delay energy consumption in time. Several classic [8], [33] and more recent [2] papers have explored the concept of reliability-differentiated pricing of interruptible electric power service, where consumers take on the risk of supply interruption in exchange for a reduction in the nominal energy price. Beyond the difficulty in auditing such markets and the apparent issues of moral hazard, the primary drawback of such approaches stems from the explicit transferal of quantity or price risk to the demand side. This amounts to requiring that consumers plan consumption in the face of uncertain supply or prices – a nontrivial task.

With the aim of alleviating the aforementioned challenges, we propose and analyze a novel forward market for *deadline differentiated energy services*,[1] where consumers consent to deferred supply of energy in exchange for a reduced per-unit energy price. Such a market would naturally complement the proliferation of plug-in electric vehicles (EV) in the US transportation fleet.

Example 1 (Electric Vehicle Charging)**.** With this motivation in mind, consider the following example describing the interaction between an EV owner and a principal responsible for administering the market – a load aggregator, for example. An EV owner arrives at her destination and connects her EV to the grid. Upon plugging in, the EV owner is presented with a menu of prices – each of which stipulates a per-unit price for energy and a corresponding delivery deadline. Faced with such choices, how does the EV owner decide upon which bundle of energy-deadline pairs to purchase given the inherent ability to delay consumption? On the supply side, the load aggregator is required to meet all energy requests by their corresponding deadlines. How should the aggregator set the menu of deadline differentiated prices to illicit the desired demand from a population of EV owners? What if the available energy supply is random? In order to address such questions, we first require a description of the underlying market, which we informally describe below to orient the reader. □

[1]The concept of deadline differentiated pricing was first proposed in a conference paper [3] and subsequently empirically evaluated in [26].

Market Operation. The forward market for deadline differentiated energy service operates according to a three-step process. In step 1, the supplier announces a mechanism. In step 2, the consumers simultaneously report their demand. In step 3, the mechanism is executed; namely, prices are set and the requested demand is delivered to each consumer. Time is assumed to be discrete with periods indexed by $k = 0, 1, 2, \ldots, N$.

Step 1 (Mechanism Design). Prior to period $k = 0$, the supplier announces a *market mechanism* $(\pi, \boldsymbol{\kappa})$ consisting of both a *scheduling policy* π (cf. its formal definition in Section III), and *pricing scheme* $\boldsymbol{\kappa} = (\kappa_1, \ldots, \kappa_N)$ that maps the aggregate demand bundle $\mathbf{x}$ (cf. its definition in Step 2) into a menu of *deadline-differentiated prices*,

$$p_k = \kappa_k(\mathbf{x}), \qquad k = 1, \ldots, N. \tag{1}$$

The price menu stipulates a per-unit price p_k (\$/kWh) for energy guaranteed delivery by period k. At the heart of the mechanism design is the restriction that prices are nonincreasing in the deadline. Namely, the longer a customer is willing to defer her consumption, the less she is required to pay. We will use $\mathcal{P} = \{\mathbf{p} \in \mathbb{R}^N_+ \mid p_1 \geq p_2 \geq \ldots \geq p_N\}$ to denote the set of feasible deadline-differentiated price bundles.

Step 2 (Consumer Reporting). Each consumer then reports a bundle of deadline-differentiated energy quantities $\mathbf{a} = (a_1, \ldots, a_N)^\top \in \mathbb{R}^N_+$. Here, the quantity a_k (kWh) denotes the amount of energy guaranteed delivery by the deadline k. It follows that said consumer will receive at least $\sum_{t=1}^k a_t$ amount of energy by deadline k. The aggregate demand bundle is the sum of all individual consumer bundles, which we denote by $\mathbf{x} = (x_1, \ldots, x_N)^\top \in \mathbb{R}^N_+$.

Step 3 (Pricing and Energy Delivery). Given an aggregate demand bundle $\mathbf{x}$, the deadline differentiated prices are set according to $\mathbf{p} = \boldsymbol{\kappa}(\mathbf{x})$. Pricing is non-discriminatory, in that all customers are charged according to the same menu of prices. Thus, a customer requesting a bundle $\mathbf{a}$ pays $\mathbf{p}^\top \mathbf{a}$. The supplier must also deliver the aggregate demand bundle $\mathbf{x}$ according to the previously announced scheduling policy π. Essentially, a scheduling policy is said to be feasible if it delivers each consumer's requested energy bundle by its corresponding deadline. The supplier is assumed to have two sources of electricity from which he can service demand: *intermittent* and *firm*.

- *Intermittent supply:* An intermittent supply modeled as a discrete time random process $\mathbf{s} = (s_0, s_1, \ldots, s_{N-1})$ with known joint probability distribution. Here, $s_k \in \mathcal{S}$ (kWh) denotes the energy produced during period k and $\mathcal{S} \subset \mathbb{R}_+$ the feasible supply interval. The intermittent supply is assumed to have zero marginal cost.
- *Firm supply.* A firm supply with a fixed price of $c_0 > 0$. The price c_0 can be interpreted as the nominal flat rate

for electricity set by the local utility.

While stylized in nature, our models of supply and demand are meant to reflect the essential features of an emerging grid infrastructure that enables the direct coupling of deferrable electric loads with variable renewable supply. Of particular relevance to our development is the growing multitude of companies offering turn-key products that integrate EV charging infrastructure with solar photovoltaic canopies [10], [28].

C. Summary and Main Contribution

A basic contribution of our paper is the design of a market mechanism for deadline differentiated energy services, which implements truth-telling at a dominant strategy equilibrium across the population of consumers, while maximizing social welfare at a competitive equilibrium between the supplier and consumers. We provide here a roadmap of the paper together with a summary of our main results.

- We provide a stylized, yet descriptive, model of both deferrable electricity demand and a supplier with both firm and intermittent supply in Sections II and III, respectively.
- We formulate the supplier's scheduling problem as a constrained stochastic optimal control problem and show average-cost optimality of the earliest-deadline-first (EDF) scheduling policy. As a corollary, this result enables the explicit characterization of the supplier's marginal cost curve in Theorem 2.
- It is reasonable to expect that the supplier cannot observe the true deadline of each individual consumer. And the presence of asymmetric information may lead to significant welfare loss, if consumers misreport their true deadlines. Somewhat surprisingly, we show under mild assumptions on consumers' utility functions that a mechanism consisting of an EDF scheduling policy, together with a uniform marginal cost pricing scheme implements consumers' *truth-telling* behavior in dominant strategies.
- We further show in Section IV-B that a marginal cost pricing, in combination with EDF scheduling, induces an *efficient competitive equilibrium* between a population of truth-telling consumers and a price-taking supplier.

D. Related Work

There have recently emerged several papers concerned with the design of incentives for delay-tolerant electric loads. The authors of [15] propose an idea similar to that of this paper, where consumers are offered a discounted electricity price in exchange for the delay of their energy consumption. However, the focus of [15] is not on pricing, but rather on the problem of optimal scheduling faced by the operator, who is assumed to have full information about consumers – for example, knowledge of their true deadlines. Closer to the present paper, [23] propose a greedy online mechanism for electric vehicle charging, which is shown to be incentive compatible and achieve a bounded (worst-case) competitive ratio. These theoretical results require, however, a VCG-type (discriminatory-price) payment scheme, as opposed to the uniform-price scheme proposed in this paper. More strongly, they impose an additional assumption that consumers cannot report false deadlines exceeding their true underlying deadlines. Such assumption substantially simplifies the problem of designing an incentive compatible pricing scheme when consumers are permitted to report arbitrary deadlines – the setting considered in the present paper.

II. Model of Demand

We consider a model involving a continuum of consumers, indexed by $i \in [0, 1]$. Since each individual consumer's action has no influence on the price, she will act as a price taker. Such an assumption is reasonable and commonly employed in the context of retail electricity markets [8], [19], [31], [33], where it is not uncommon for an electric power utility to service 10^4 to 10^6 customers – a setting in which each consumer herself is too small to influence the price.

We propose a consumer utility model yielding a preference ordering on deadlines, where the longer the delay in consumption, the smaller the utility derived from consumption. In particular, we assume that each consumer has a single deadline preference. More precisely, a consumer with deadline preference k incurs no loss of utility by deferring consumption until deadline k and derives zero utility for any consumption thereafter. This assumption is reasonable for electric loads such as plug-in electric vehicles (PEVs), dish washers, and laundry machines, as customers commonly require only that such loads fully execute before a specific time. Such intuition lends itself to the following definition of *consumer type*.

Definition 1 (Consumer type)**.** The *type of consumer* i is a triple, $\theta_i = (k_i, R_i, q_i)$, consisting of her *deadline* $k_i \in \mathbb{N}$, *marginal utility* $R_i \in \mathbb{R}_+$, and *maximum demand* $q_i \in \mathbb{R}_+$.

Consumer i's utility function depends only on her type θ_i and is assumed to satisfy the following conditions.

Assumption 1. A consumer of type θ_i derives a utility that depends only on the total energy consumed by her true deadline k_i. The *utility function* $U_{\theta_i} : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed to be non-negative and non-decreasing over $[0, q_i]$ with

$$U_{\theta_i}(y) \le yR_i, \quad \text{for all} \quad y \in [0, q_i],$$

where $R_i = U_{\theta_i}(q_i)/q_i$. We also assume that $U_{\theta_i}(y) = U_{\theta_i}(q_i)$ for all $y \ge q_i$. □

Remark 1. Note that the marginal utility R_i associated with a consumer type θ_i is defined as the ratio of the maximum utility $U_{\theta_i}(q_i)$ to the maximum demand q_i. It is also worth emphasizing that Assumption 1 accommodates a large family of utility functions. Indeed, every utility function that lies below the piecewise affine function $U_{\theta_i}(y) = R_i \min\{y, q_i\}$ satisfies Assumption 1. Figure 1 depicts several such utility functions. The utility function depicted in Figure 1(b) characterizes electric loads with with 'all or nothing' utility characteristics. For example, in the context of electric vehicle charging, $q_i > 0$ can be interpreted as the minimum amount of battery charge required by the consumer in order to fulfill her next trip. Naturally, the consumer obtains zero utility if the battery level

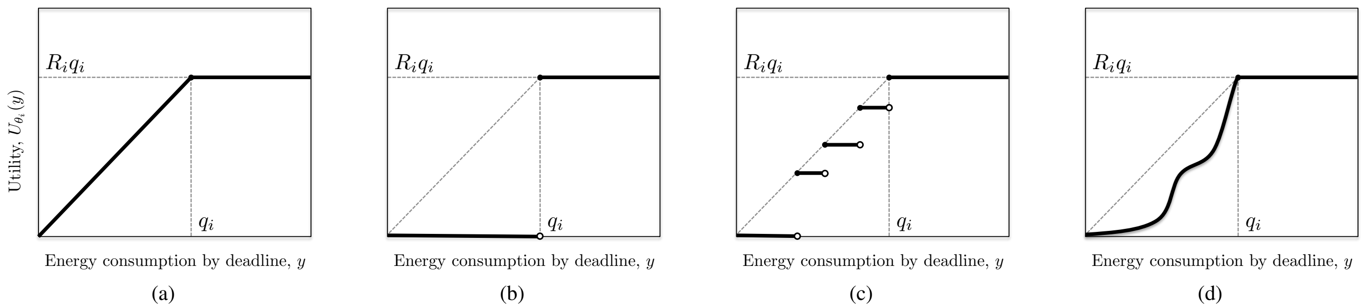

Fig. 1. Four examples of utility functions satisfying Assumption 1.

is below this threshold. Many job-oriented appliances, such as dishwashers, are similarly modeled. The utility function in Figure 1(c) is a natural generalization of that in 1(b), and could capture the utility of a consumer having multiple all-or-nothing jobs requiring completion before a common deadline. Finally, the utility function in Figure 1(d) is a general non-convex utility function that satisfies Assumption 1. It is also worth mentioning that Assumption 1 prevents the treatment of general concave utility functions, as incentive compatibility may fail to hold for certain concave utility functions. We refer the reader to Remark 3 for a more detailed discussion.

We let Θ denote the *set of all possible consumer types*, which is assumed to be finite. Let $\rho : \Theta \to [0,1]$ denote the *distribution of consumer types* over the space Θ. In other words, for every $\theta \in \Theta$, there is a $\rho(\theta)$ fraction of consumers of type θ. It follows that $\sum_{\theta\in\Theta} \rho(\theta) = 1$.

Definition 2 (Consumer action)**.** The *action* of a consumer is a vector $\mathbf{a} = (a_1, \ldots, a_N)^\top \in \mathbb{R}_+^N$, where a_k denotes the amount of energy that is guaranteed delivery by deadline k. The maximum amount of energy any consumer can request is $Q = \max_{(k,R,q)\in\Theta}\{q\}$. Hence, each consumer's *action space* is restricted to $\mathcal{A} = \left\{\mathbf{a} \in \mathbb{R}_+^N \mid \sum_k a_k \le Q\right\}$.

It follows from the above definition that $q \le Q$ for every type $\theta = (k, R, q)$. In other words, it is feasible for every consumer to request her maximum demand q. Given a fixed scheduling policy and pricing scheme, a consumer's *strategy* $\varphi : \Theta \to \mathcal{A}$ maps her type into an action. In the proceeding analysis, we will be concerned with identifying conditions on both the scheduling policy and pricing scheme that lead to efficient allocations, while inducing consumers to truthfully reveal their underlying deadline preferences. A consumer of type $\theta = (k, R, q)$ is defined to be *truth-telling* if she requests q units of energy at her true deadline k, and nothing else. We make this notion precise in the following definition.

Definition 3 (Truth-telling)**.** A consumer of type $\theta = (k, R, q)$ is defined to be *truth-telling* if her strategy $\mathbf{a}^* = \varphi^*(\theta)$ satisfies $a_j^* = 0$ for all $j \ne k$ and $a_k^* = q$.

We emphasize that under an arbitrary scheduling policy and pricing scheme, it is indeed possible that a consumer's best response is untruthful. Our aim is to provide an explicit characterization of a scheduling policy and pricing scheme that implement truth-telling as a dominant strategy for every consumer i. Given the collection of consumer types $\boldsymbol{\theta} = \{\theta_i\}_{i\in[0,1]}$ and a strategy profile $\boldsymbol{\varphi} = \{\varphi_i\}_{i\in[0,1]}$, the aggregate demand bundle $\mathbf{x}$ is given by the mapping

$$\mathbf{x} = \mathbf{d}(\boldsymbol{\theta}, \boldsymbol{\varphi}) = \int_{i\in[0,1]} \varphi_i(\theta_i)\eta(di), \tag{2}$$

where η is the Lebesgue measure defined over $[0,1]$, and $\mathbf{d} = (d_1, \ldots, d_N)$ maps $(\boldsymbol{\theta}, \boldsymbol{\varphi})$ into an N-dimensional non-negative vector.[2] Under the *truth-telling strategy profile* $\boldsymbol{\varphi}^* = \{\varphi_i^*\}_{i\in[0,1]}$ specified in Definition 3, the aggregate demand bundle simplifies to

$$x_j^* = d_j(\boldsymbol{\theta}, \boldsymbol{\varphi}^*) = \sum_{\theta\in\Theta} q \cdot \rho(\theta) \cdot \mathbf{1}_{\{j=k\}} \tag{3}$$

for all $j = 1, \ldots, N$, where $\theta = (k, R, q)$. Here, $\mathbf{1}_{\{\cdot\}}$ denotes the indicator function for the event in the subscript.

A. Consumer Surplus

We are now in a position to characterize the expected surplus derived by a consumer. It depends on: (i) her own type and strategy, (ii) the remaining consumers' types and strategy profile, (iii) the pricing scheme, and of course, (iv) the scheduling policy employed by the supplier. Before proceeding, we require the definition of pertinent notation. We define the random variable $\omega_{k,i}^\pi(\mathbf{x}, \mathbf{a})$ to denote the amount of energy delivered to consumer i by stage k given her requested bundle $\mathbf{a}$ and aggregate demand bundle $\mathbf{x}$. This random variable, which naturally depends on the scheduling policy π employed by the supplier, is formally defined in Equation (16) in Appendix B.[3]

We require that the requested quantities are always supplied by their corresponding deadlines and the total quantity delivered to a consumer never exceeds said consumer's total demand. More formally, we require for each consumer $i \in [0,1]$ taking action $\mathbf{a} \in \mathcal{A}$ that

$$\sum\nolimits_{t=1}^{k} a_t \;\le\; \omega_{k,i}^\pi(\mathbf{x}, \mathbf{a}) \;\le\; \sum\nolimits_{t=1}^{N} a_t \tag{4}$$

with probability one, for all aggregate demand bundles $\mathbf{x}$ and $k = 1, \ldots, N$.

In order to formally define and analyze incentive compatibility of our proposed market mechanism, we require a

[2]Note that we have implicitly assumed that for every $\boldsymbol{\theta}$, the function $\boldsymbol{\varphi} = \{\varphi_i(\theta_i)\}_{i\in[0,1]}$ is Lebesgue integrable in i. This assumption holds, for example, under a symmetric strategy profile according to which all consumers of the same type take the same action.

[3]Note that we have allowed the supply $\omega_{k,i}^\pi(\mathbf{x}, \mathbf{a})$ to depend explicitly on the consumer index i, as the supplier may employ a scheduling policy that depends on the consumer index.

definition of the expected surplus derived by a consumer under a particular strategy. Given a scheduling policy π and a pricing scheme $\boldsymbol{\kappa}$ employed by the supplier, and all consumer types $\boldsymbol{\theta} = \{\theta_i\}_{i\in[0,1]}$, consumer i receives an *expected surplus* (payoff) under a strategy profile $\boldsymbol{\varphi} = \{\varphi_i\}_{i\in[0,1]}$ that is given by

$$v_i^\pi(\boldsymbol{\theta}, \boldsymbol{\varphi}, \boldsymbol{\kappa}) \; = \; \mathbb{E}\left\{U_{\theta_i}\left(\omega^\pi_{k_i,i}(\mathbf{x}, \mathbf{a})\right)\right\} \; - \; \boldsymbol{\kappa}(\mathbf{x})^\top \mathbf{a}. \quad (5)$$

Here, k_i is consumer i's true deadline, $\mathbf{x} = \mathbf{d}(\boldsymbol{\theta}, \boldsymbol{\varphi})$ is the aggregate demand bundle, $\mathbf{a} = \varphi_i(\theta_i)$ is the action taken by consumer i, and $\boldsymbol{\kappa}(\mathbf{x})$ is the price bundle set according to the pricing scheme $\boldsymbol{\kappa}$ at the aggregate demand $\mathbf{x}$. Expectation is taken with respect to the random variable $\omega^\pi_{k_i,i}(\mathbf{x}, \mathbf{a})$.

Clearly, a scheduling policy π together with a pricing scheme $\boldsymbol{\kappa}$ defines a game for the consumer population, with each individual consumer's payoff expressed in (5). We note that this is an *aggregative game*, where the payoff function of each player depends on the population's strategy profile only through the sum of their actions – the aggregate demand bundle $\mathbf{x} = \mathbf{d}(\boldsymbol{\theta}, \boldsymbol{\varphi})$. We can therefore rewrite consumer i's payoff in (5) in a form that depends on other players' strategies only through their sum $\mathbf{x}$. Namely,

$$V_i^\pi(\theta_i, \varphi_i, \mathbf{x}, \boldsymbol{\kappa}) = \; \mathbb{E}\left\{U_{\theta_i}\left(\omega^\pi_{k_i,i}(\mathbf{x}, \mathbf{a})\right)\right\} \; - \; \boldsymbol{\kappa}(\mathbf{x})^\top \mathbf{a}, \quad (6)$$

where $\mathbf{a} = \varphi_i(\theta_i)$. Moreover, it follows from the service constraint in (4) that under the truth-telling strategy φ_i^*, the payoff derived by consumer i simplifies to the deterministic quantity

$$V_i^\pi(\theta_i, \varphi_i^*, \mathbf{x}, \boldsymbol{\kappa}) \; = \; U_{\theta_i}(q_i) \; - \; \kappa_{k_i}(\mathbf{x}) q_i, \quad (7)$$

where $\theta_i = (k_i, R_i, q_i)$. It is important to note that the expression in (7) does not depend on the types and strategies of the other consumers, or the scheduling policy used by the supplier, as long as the service constraint in (4) is respected.

Bayesian Nash equilibrium may not be a plausible solution concept to explore for this game, as it requires each individual consumer to have information regarding the distribution of other consumers' types, as well as knowledge of the probability distribution of $\omega^\pi_{k_i,i}(\mathbf{x}, \mathbf{a})$, which in turn depends on the distribution of the intermittent supply process $\mathbf{s}$. We circumvent such informational assumptions by focusing our analysis around a stronger solution concept – namely, the dominant strategy equilibrium of the game. A strategy φ_i is a *dominant strategy* for consumer i of type θ_i, if it maximizes her expected payoff regardless of the actions taken by the other consumers. We have the following definition.

Definition 4 (Dominant strategy)**.** A strategy φ_i is a *dominant strategy* for a consumer i of type θ_i if

$$V_i^\pi(\theta_i, \varphi_i, \mathbf{x}, \boldsymbol{\kappa}) \; \geq \; V_i^\pi(\theta_i, \varphi_i', \mathbf{x}, \boldsymbol{\kappa}), \quad \forall \; \varphi_i', \quad \forall \; \mathbf{x} \in \mathbb{R}^N_+.$$

In Definition 6, we define a mechanism to be *incentive compatible* if it implements truth-telling in dominant strategies. Surprisingly, we show in Section IV that a mechanism consisting of an earliest-deadline-first (EDF) scheduling policy in combination with a marginal cost pricing scheme is incentive compatible and induces an efficient competitive equilibrium between the supplier and consumers.

III. Model of Supply

As one of the primary thrusts of this paper is the characterization of a competitive equilibrium between the supplier and consumers and its efficiency properties, we now consider the behavior of a *price-taking supplier*, whose aim is to maximize his expected profit given a predetermined price bundle. The expected profit derived by a supplier equals the revenue derived from the sale of a bundle of deadline differentiated energy quantities less the expected cost of firm supply required to service said bundle. In determining his supply curve under price taking behavior, the supplier's objectives are two-fold:

- *Scheduling.* Determine an *optimal scheduling policy* to causally allocate the intermittent supply across the deadline differentiated consumer classes, in order to minimize the expected cost of firm supply required to ensure that demand bundles are served by their respective deadlines.
- *Pricing.* Given the optimal scheduling policy, determine a *marginal cost supply curve* that specifies the bundle of energy he is willing to supply at every price bundle.

The problem of characterizing a marginal cost supply curve amounts to the explicit solution of a two-stage stochastic program, whose expected recourse cost is the optimal value of a constrained scheduling problem parameterized by the aggregate demand bundle. In Theorem 2, we explicitly characterize a supply curve, which is used to construct an efficient competitive equilibrium between supply and demand.

A. *Optimal Scheduling Policy*

We now offer an abbreviated formulation of supplier's scheduling problem. A detailed formulation is presented in Appendix A. When considering the problem of scheduling, it is important to distinguish between intra-class and inter-class scheduling. An *inter-class scheduling policy* (denoted by σ) represents a sequence of scheduling decisions, which causally allocate the intermittent and firm supply across the deadline-differentiated demand classes. We denote by $\Sigma(\mathbf{x})$ the space of all *feasible inter-class scheduling policies* available for use by the supplier, given an aggregate demand bundle $\mathbf{x}$. In addition to inter-class scheduling, the supplier must also determine as to how the available supply is allocated between customers within a given demand class. As such, we let the *intra-class scheduling policy* (denoted by ϕ) represent a sequence of scheduling decisions, which dictate how the intermittent and firm supply made available to a given demand class is allocated across customers within said class. We denote by $\Phi(\sigma)$ the space of all *feasible intra-class scheduling policies* available for use by the supplier, which naturally depends on the inter-class scheduling policy σ being used. Finally, we let $\pi = (\sigma, \phi)$ denote the *joint inter-class and intra-class scheduling policy* employed by the supplier.

It is important to note that the supplier's expected profit depends only on the inter-class scheduling policy being used. This follows from the supplier's indifference to supply allocation between consumers within a given demand class. Therefore, in characterizing the optimal scheduling policy for the supplier, we restrict our attention to the characterization of optimal inter-class policies for the remainder of the paper.[4]

We define the *expected profit* $J(\mathbf{x}, \mathbf{p}, \sigma)$ derived by a supplier as the revenue derived from an aggregate demand bundle $\mathbf{x}$ less the expected cost of servicing said demand bundle under a feasible inter-class scheduling policy $\sigma \in \Sigma(\mathbf{x})$. More precisely, let

$$J(\mathbf{x}, \mathbf{p}, \sigma) \; = \; \mathbf{p}^\top \mathbf{x} \; - \; Q(\mathbf{x}, \sigma), \tag{8}$$

where Q denotes the expected cost of firm generation incurred servicing $\mathbf{x}$ under a feasible policy $\sigma \in \Sigma(\mathbf{x})$. We wish to characterize scheduling policies that lead to a minimal expected cost of firm supply.

Definition 5 (Optimal Policy)**.** The inter-class scheduling policy $\sigma^* \in \Sigma(\mathbf{x})$ is defined to be *optimal* if

$$Q(\mathbf{x}, \sigma^*) \; \leq \; Q(\mathbf{x}, \sigma), \qquad \text{for all} \;\; \sigma \in \Sigma(\mathbf{x}). \tag{9}$$

We denote by $Q^*(\mathbf{x}) = Q(\mathbf{x}, \sigma^*)$ the minimum expected cost of firm supply.

The following result provides an explicit characterization of an optimal inter-class scheduling policy.

Theorem 1 (Earliest-Deadline-First)**.** Let $\mathbf{x} \in \mathbb{R}^N_+$. The inter-class scheduling policy $\sigma^* \in \Sigma(\mathbf{x})$ is optimal if *(i)* the intermittent supply s_k available at each period k is allocated to those unsatisfied demand classes with *earliest-deadline-first* (EDF), and *(ii)* the firm supply is allocated to a demand class only when the EDF allocation of intermittent supply is insufficient to ensure its deadline satisfaction.[5]

For the remainder of the paper, we refer to σ^* as the *EDF scheduling policy.* The EDF scheduling policy is attractive from an implementation perspective; it is *distribution-free*. As such, the optimal schedule can be computed causally without requiring explicit knowledge of the probability distribution on the intermittent supply process.

B. Marginal Cost Pricing

Given the EDF characterization of the optimal inter-class scheduling policy in Theorem 1, we are now in a position to characterize the supplier's optimal supply curve under price taking behavior. We define the *residual process* induced by the EDF scheduling policy σ^* as

$$\xi_{k+1}(\mathbf{x}, \mathbf{s}) \; = \; \max\{0, \; \xi_k(\mathbf{x}, \mathbf{s})\} \; + \; s_k - x_{k+1}, \tag{10}$$

for $k = 0, \ldots, N-1$, where $\xi_0 = 0$. We denote the entire residual process by $\boldsymbol{\xi} = (\xi_0, \ldots, \xi_N)$, omitting its dependency on $\mathbf{x}$ and $\mathbf{s}$ when it is clear from the context. A positive residual ($\xi_k > 0$) represents the amount of intermittent supply leftover after having serviced demand class k by its deadline k, according to the EDF inter-class scheduling policy σ^*. A negative residual ($\xi_k \leq 0$) represents the amount by which the intermittent supply fell short – or, equivalently, the amount of firm supply required to ensure satisfaction of the demand class k. Using this newly defined process, we have the following characterization of the minimum expected cost of firm supply under EDF scheduling. First, we require a technical assumption.

Assumption 2. The joint probability distribution of the intermittent supply process $\mathbf{s}$ is assumed to be absolutely continuous and have compact support. □

Lemma 1. Suppose that Assumption 2 holds. The minimum expected recourse cost $Q^*(\mathbf{x})$ derived under an aggregate demand bundle $\mathbf{x} \in \mathbb{R}^N_+$ and EDF scheduling policy $\sigma^* \in \Sigma(\mathbf{x})$ satisfies

$$Q^*(\mathbf{x}) \; = \; \mathbb{E}\left\{ c_0 \sum_{k=1}^{N} - \min\{0, \; \xi_k\} \right\}, \tag{11}$$

and is convex and differentiable in $\mathbf{x}$ over $[0, \infty)^N$.

Lemma 1, the proof of which can be found in Appendix C, admits a natural interpretation. Namely, the minimum expected cost of firm supply is equivalent to the amount by which the intermittent supply is expected to fall short for each demand class under EDF scheduling. Moreover, it follows readily that the expected profit $J(\mathbf{x}, \mathbf{p}, \sigma^*)$ derived under EDF scheduling is also differentiable and concave in $\mathbf{x}$. As such, any allocation $\mathbf{x}$ satisfying the first order condition,

$$\nabla_{\mathbf{x}} \, J(\mathbf{x}, \mathbf{p}, \sigma^*)^\top (\mathbf{x} - \mathbf{y}) \; \geq \; 0 \qquad \text{for all} \quad \mathbf{y} \in \mathbb{R}^N_+, \tag{12}$$

is profit maximizing for the supplier given a price bundle $\mathbf{p} \in \mathcal{P}$. Accordingly, we provide an explicit characterization of the supplier's *marginal cost supply curve* in the following theorem.

Theorem 2 (Marginal cost supply curve)**.** Suppose that Assumption 2 holds. An allocation $\mathbf{x}$ is profit maximizing for a given price bundle $\mathbf{p}$, if $\mathbf{p} = \boldsymbol{\zeta}(\mathbf{x})$, where the mapping $\boldsymbol{\zeta} : \mathbb{R}^N_+ \to \mathcal{P}$ satisfies, for $k = 1, \ldots, N$,

$$\frac{\zeta_k(\mathbf{x})}{c_0} = \mathbb{P}(\xi_k \leq 0) + \sum_{t=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{t-1} > 0, \xi_t \leq 0). \tag{13}$$

The *marginal cost pricing scheme* specified in Equation (13) maps every aggregate demand bundle in $\mathbb{R}^N_+$ into a menu of deadline differentiated price in $\mathcal{P}$. Moreover, one can readily interpret such pricing scheme as setting the price p_k for energy with deadline k equal to (up to a proportionality constant c_0) the probability that firm supply will be required to service the bundle $\mathbf{x}$ at any subsequent time period $t \geq k-1$ under the optimal inter-class scheduling policy σ^*. Naturally, the larger the probability of shortfall, the larger the price. It is readily verified that the pricing scheme $\mathbf{p} = \boldsymbol{\zeta}(\mathbf{x})$ yields, in general, prices that are nonincreasing in the deadline. More precisely,

$$c_0 \; \geq \; p_1 \; \geq \; p_2 \; \geq \cdots \geq \; p_N$$

for all $\mathbf{x} \in \mathbb{R}^N_+$. This property of price monotonicity is consistent with our initial criterion for constructing such a market system. Namely, the longer a customer is willing to

[4] The distinction between inter-class and intra-class scheduling is however important, as the choice of intra-class scheduling policy ϕ can influence consumer purchase decisions as it affects the probability distribution on each consumer's random supply $\omega^{\pi}_{k_i,i}(\mathbf{x}, \mathbf{a})$. A formal definition of feasible intra-class policies is developed in Appendix B.

[5] A more formal statement of Theorem 1 can be found in Appendix A.

defer her consumption in time, the less she is required to pay per unit of energy. And the price of deferrable energy is no greater than the nominal flat rate for electricity, c_0.

IV. Incentive Compatibility and Efficiency

We now establish several important properties of the market mechanism developed in Section III. In particular, under an additional mild assumption on each consumer's marginal valuation on energy, we show in Sections IV-A and IV-B that a mechanism consisting of earliest-deadline-first (EDF) scheduling in combination with the marginal cost pricing scheme in (13) is indeed *incentive compatible* and *achieves social optimality* at a competitive market equilibrium, respectively.

A. Incentive Compatibility

With market efficiency considerations in mind, it is important to understand when a consumer has incentive to misreport her underlying deadline preference. A mechanism $(\pi, \boldsymbol{\kappa})$ consisting of a feasible scheduling policy $\pi = (\sigma, \phi)$ and pricing scheme $\boldsymbol{\kappa}$ is said to be *incentive compatible* (IC) for consumers of a particular type, if it is a dominant-strategy for consumers of this type to be truth-telling.

Definition 6 (Incentive compatibility)**.** A mechanism $(\pi, \boldsymbol{\kappa})$ is incentive compatible for every consumer i of type θ_i, if

$$V_i^{\pi}(\theta_i, \varphi_i^*, \mathbf{x}, \boldsymbol{\kappa}) \;\geq\; V_i^{\pi}(\theta_i, \varphi_i, \mathbf{x}, \boldsymbol{\kappa}), \quad \forall\, \varphi_i, \;\; \forall\, \mathbf{x} \in \mathbb{R}_+^N,$$

where φ_i^* is the truth-telling strategy defined in Definition 3.

It is worth mentioning that the above definition of dominant strategy incentive compatibility is strong. It requires that a consumer would like to reveal her true type regardless of the types and actions of other consumers and the probability distribution on the intermittent supply process. Since the optimal price schedule is non-increasing in the deadline, and demand is guaranteed to be met before the requested deadline, a consumer i does not have an incentive to request any quantity of energy before her true deadline k_i. However, if the price of energy associated with later deadlines in low enough, said consumer may have an incentive to report a false later deadline if early delivery is likely (i.e., with high probability) under the specified scheduling policy. Intuitively, a consumer i will have incentive to misreport its deadline if the reduction in total expenditure derived by requesting energy with later deadlines exceeds the expected loss of utility incurred by a shortfall in the amount of energy delivered by her true deadline k_i. Surprisingly, we show in Theorem 3 that EDF scheduling in combination with marginal cost pricing precludes this possibility.

For the remainder of the paper, we denote by $(\pi^*, \boldsymbol{\zeta})$ the market mechanism defined by EDF scheduling and marginal cost pricing. More precisely, the scheduling policy $\pi^* = (\sigma^*, \phi)$ consists of the EDF inter-class scheduling policy σ^* and an arbitrary feasible intra-class policy $\phi \in \Phi(\sigma^*)$. And, $\boldsymbol{\zeta}$ denotes the marginal cost pricing scheme in Equation (13).

Theorem 3 (Incentive compatibility)**.** Suppose that Assumptions 1-2 hold. The mechanism $(\pi^*, \boldsymbol{\zeta})$ is incentive compatible for all consumers of a type that satisfies $R \geq c_0$.

We further establish in Corollary 1 that the mechanism $(\pi^*, \boldsymbol{\zeta})$ also maximizes social welfare at a unique market equilibrium between the supplier and consumers, if the condition $R \geq c_0$ holds for every consumer type $\theta \in \Theta$.

Remark 2. We note that the requirement $R \geq c_0$ is reasonable for electricity consumers, as their marginal valuation on electricity consumption is commonly higher than the nominal flat rate for electricity, which in our model is denoted by c_0. On this basis, electricity demand is generally modeled as inelastic [30], [34], especially in the short term [37].

Remark 3. We also note that incentive compatibility may fail to hold if certain assumptions regarding a consumer's utility function are violated. First, one can readily show that incentive compatibility may fail to hold for a consumer i of type $\theta_i = (k_i, R_i, q_i)$, if the marginal cost of firm supply exceeds her marginal valuation of energy – namely, $R_i < c_0$. Second, we note that the result in Theorem 3 fails to hold for arbitrary concave utility functions. This is intuitive. Consider a consumer i having a highly concave utility function. Because of the large underlying concavity in her utility, said consumer will prefer to report a false deadline that is later than her true deadline k_i, if she can obtain a fraction of her demand before stage k_i with high probability and at a low price.

B. Market Equilibrium and Efficiency

In this section, we show that mechanism consisting of marginal cost pricing together with EDF scheduling, results in an efficient market equilibrium at which social welfare (the sum of aggregate consumer surplus and supplier profit) is maximized. First, we make an assumption under which we will operate for the remainder of the paper.

Assumption 3. We assume that every consumer type $(k, R, q) \in \Theta$ satisfies $R \geq c_0$. ☐

See Remark 2 for a discussion on Assumption 3. It follows from Assumption 3 and Theorem 3 that under the mechanism $(\pi^*, \boldsymbol{\zeta})$, it is a dominant strategy for every consumer to be truth-telling. The aggregate demand resulting from the true-telling population, which we denote by $\mathbf{x}^*$, is given by Equation (3). Theorem 2 shows that it is profit-maximizing for a supplier to meet the aggregate demand $\mathbf{x}^*$ at the price bundle $\mathbf{p} = \boldsymbol{\zeta}(\mathbf{x}^*)$. In what follows, we will show that resulting quantity-price pair $(\mathbf{x}^*, \boldsymbol{\zeta}(\mathbf{x}^*))$ constitutes a market equilibrium that is social welfare maximizing. We first offer a definition of *market equilibrium.*

Definition 7 (Market equilibrium)**.** Let $(\pi, \boldsymbol{\kappa})$ be a market mechanism consisting of a scheduling policy π and pricing scheme $\boldsymbol{\kappa}$. Given the types of all consumers $\boldsymbol{\theta}$, a quantity-price pair $(\mathbf{x}, \mathbf{p})$ is a *market equilibrium* under the mechanism $(\pi, \boldsymbol{\kappa})$ if the following two conditions hold.

(i) It is a dominant strategy for every consumer to be truth-telling under the mechanism $(\pi, \boldsymbol{\kappa})$, and the resulting aggregate demand bundle is $\mathbf{x}$.

(ii) The aggregate demand bundle $\mathbf{x}$ together with the scheduling policy π maximize a price-taking supplier's expected profit at price bundle $\mathbf{p} = \boldsymbol{\kappa}(\mathbf{x})$.

In Definition 7, the first condition ensures that under the mechanism $(\pi, \boldsymbol{\kappa})$, the aggregate demand (resulting from a truth-telling non-atomic population) is $\mathbf{x}$. The the second condition requires that given the price bundle $\mathbf{p} = \boldsymbol{\kappa}(\mathbf{x})$ (which is determined according to the pricing scheme $\boldsymbol{\kappa}$ and aggregate demand $\mathbf{x}$), a price-taking supplier would like to employ the scheduling policy π and supply the bundle $\mathbf{x}$. We therefore have supply equal demand. We show in the following corollary that the EDF scheduling policy $\pi^* = (\sigma^*, \phi)$ together with the marginal cost pricing scheme $\boldsymbol{\zeta}$ constitute a market mechanism that induces a unique market equilibrium, which maximizes the social welfare.

Corollary 1. Suppose that Assumptions 1-3 hold. Given the types of all consumers $\boldsymbol{\theta}$ and under a mechanism $(\pi^*, \boldsymbol{\zeta})$, there exists a unique market equilibrium $(\mathbf{x}^*, \boldsymbol{\zeta}(\mathbf{x}^*))$ that maximizes the social welfare. Here, $\mathbf{x}^*$ is the truth-telling aggregate demand bundle specified in Equation (3).

V. Conclusion

To explore the flexibility of delay-tolerant electricity loads, we propose a novel deadline-differentiated market for energy that offers discounted (per-unit) electricity prices to consumers in exchange for their consent to defer their electric power consumption, and provides a guarantee on the aggregate quantity of energy to be delivered by a consumer-specified deadline. We provide a full characterization of the joint scheduling and pricing scheme that yields an efficient (competitive) market equilibrium between a (price-taking) supplier and a consumer population. Somewhat surprisingly, we show that this efficient mechanism is incentive compatible in that every consumer would like to reveal her true deadline to the supplier, regardless of the actions taken by other consumers.

The market we have considered in our analysis is single shot. As a natural extension, it would be of interest to explore the dynamic analog of our formulation in which the market is cleared on a recurrent basis. In addition, such a dynamic setting could provide the foundation on which to explore efficient price discovery schemes.

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Online Supplement

Appendix A
The Supplier's Scheduling Problem

A. Inter-class scheduling policies

We characterize the optimal inter-class scheduling policy as a solution to a constrained stochastic optimal control problem. First, we define the system *state* at period k as the pair $(\mathbf{z}_k, s_k) \in \mathbb{R}_+^N \times \mathbb{R}_+$, where the vector $\mathbf{z}_k$ denotes the residual demand requirement of the original aggregate demand bundle $\mathbf{x}$ after having been serviced in previous periods $0, 1, \cdots, k-1$. Define as the *control input* the vectors $\mathbf{u}_k, \mathbf{v}_k \in \mathbb{R}_+^N$, which denote (element-wise in j) the amount of intermittent and firm supply allocated to demand class j at period k, respectively. Naturally then, the state of residual demand evolves according to the discrete time *state equation*:

$$\mathbf{z}_{k+1} \;=\; \mathbf{z}_k - \mathbf{u}_k - \mathbf{v}_k, \qquad k = 0, \ldots, N-1, \tag{14}$$

where the process is initialized with $\mathbf{z}_0 = \mathbf{x}$. The delivery deadline constraints manifest in a sequence of nested constraint sets $\mathbb{R}_+^N \supset \mathcal{Z}_1 \supseteq \mathcal{Z}_2 \supseteq \cdots \supseteq \mathcal{Z}_N = 0$ converging to the the origin, where the set $\mathcal{Z}_k$ characterizes the *feasible state space* at stage k. More precisely,

$$\mathcal{Z}_k \;=\; \{\mathbf{z} \in \mathbb{R}_+^N \mid z^j = 0, \;\; \forall\, j \leq k\}.$$

In other words, the feasible state space is such that each demand class is fully serviced by its corresponding deadline. We define as the *feasible input space* at stage k the set of all inputs belonging to the set

$$\mathcal{U}_k(\mathbf{z}, s) = \{(\mathbf{u}, \mathbf{v}) \mid \mathbf{1}^\top \mathbf{u} \leq s \text{ and } \mathbf{z} - \mathbf{u} - \mathbf{v} \in \mathcal{Z}_{k+1}\},$$

which ensures one-step state feasibility and that the total allocation of renewable supply does not exceed availability at the current stage. In characterizing the feasible set of causal scheduling policies, we restrict our attention to those policies with *Markovian information structure*, as opposed to allowing the control to depend on the entire history. This is without loss of optimality, since Markovian policies are capable of performing as well as the optimal oracle policy. We describe the scheduling decision at each stage k by the functions

$$\mathbf{u}_k = \boldsymbol{\mu}_k(\mathbf{z}, s) \quad \text{and} \quad \mathbf{v}_k = \boldsymbol{\nu}_k(\mathbf{z}, s),$$

where $\boldsymbol{\mu}_k : \mathcal{Z}_k \times \mathcal{S} \to \mathbb{R}_+^N$ and $\boldsymbol{\nu}_k : \mathcal{Z}_k \times \mathcal{S} \to \mathbb{R}_+^N$. A *feasible inter-class scheduling policy* is any finite sequence of scheduling decision functions $\sigma = (\boldsymbol{\mu}_0, \ldots, \boldsymbol{\mu}_{N-1}, \boldsymbol{\nu}_0, \ldots, \boldsymbol{\nu}_{N-1})$ such that $(\boldsymbol{\mu}_k, \boldsymbol{\nu}_k)(\mathbf{z}, s) \in \mathcal{U}_k(\mathbf{z}, s)$ for all $(\mathbf{z}, s) \in \mathcal{Z}_k \times \mathcal{S}$ and $k = 0, \ldots, N-1$. We denote by $\Sigma(\mathbf{x})$ the *feasible inter-class policy space*. Throughout the paper, we will suppress the explicit dependency of the feasible policy set on the aggregate demand bundle $\mathbf{x}$, when it is clear from the context.

An important assumption we will make on the supply side is the absence of an upper bound on the amount of energy the supplier can deliver to a customer within any given time period. Although appearing strong at first glance, such an assumption is not far from practice, as batteries with high power to energy ratios are rapidly becoming available for electric vehicles. For example, the lithium-ion titanate batteries are capable of recharging to 95% of full capacity within approximately ten minutes [4], [11]. We also note that fast (DC) chargers that can fully charge an electric vehicle within half an hour are being installed in a variety of public locations including parking lots, shopping centers, hotels, theaters, and restaurants [36].

B. Intra-class scheduling policies

Recall that an intra-class scheduling policy ϕ determines the allocation of available supply within each deadline-differentiated demand class, where the supply available to each demand class is determined by the inter-class policy $\sigma \in \Sigma$. We denote the *feasible intra-class policy space* by $\Phi(\sigma)$, which is parameterized by a given inter-class policy $\sigma \in \Sigma$. We denote by $\pi = (\sigma, \phi)$ the joint inter and intra-class scheduling policy employed by the supplier.

C. Supplier profit

We define the *expected profit* $J(\mathbf{x}, \mathbf{p}, \sigma)$ derived by a supplier as the revenue derived from an aggregate demand bundle $\mathbf{x}$ less the expected cost of servicing said demand bundle under a feasible inter-class scheduling policy $\sigma \in \Sigma(\mathbf{x})$. More precisely, let

$$J(\mathbf{x}, \mathbf{p}, \sigma) \;=\; \mathbf{p}^\top \mathbf{x} \;-\; Q(\mathbf{x}, \sigma),$$

where Q denotes the expected cost of firm generation incurred servicing $\mathbf{x}$ under a feasible policy $\sigma \in \Sigma$. It follows that

$$Q(\mathbf{x}, \sigma) \;=\; \sum_{k=0}^{N-1} \mathbb{E}\left\{\mathbf{c}^\top \mathbf{v}_k^\sigma\right\}, \tag{15}$$

where $\mathbf{c} = (c_0, \dots, c_0)$. We write the state and control process as $\{\mathbf{z}_k^\sigma\}$, $\{\mathbf{u}_k^\sigma\}$, and $\{\mathbf{v}_k^\sigma\}$ to emphasize their dependence on the policy σ. We wish to characterize scheduling policies that lead to a minimal expected cost of firm supply. The inter-class scheduling policy $\sigma^* \in \Sigma(\mathbf{x})$ is *optimal* if

$$Q(\mathbf{x}, \sigma^*) \;\le\; Q(\mathbf{x}, \sigma), \qquad \text{for all} \;\; \sigma \in \Sigma(\mathbf{x}).$$

We denote by $Q^*(\mathbf{x}) = Q(\mathbf{x}, \sigma^*)$ the minimum expected cost of firm supply. We offer a formal characterization of the optimal inter-class scheduling policy, σ^*.

D. Optimal scheduling policy

Theorem 4 (Earliest-Deadline-First)**.** Given an aggregate demand bundle $\mathbf{x} \in \mathbb{R}_+^N$, the optimal scheduling policy $\sigma^* \in \Sigma(\mathbf{x})$ is given by:

$$\mu_k^{j,*}(\mathbf{z}, s) = \min\left\{z^j, s - \textstyle\sum_{i=1}^{j-1} \mu_k^{i,*}(\mathbf{z}, s)\right\},$$
$$\nu_k^{j,*}(\mathbf{z}, s) = (z^j - \mu_k^{j,*}(\mathbf{z}, s)) \cdot \mathbf{1}_{\{k=j-1\}},$$

for $j = 1, \dots, N$, $k = 0, \dots, N-1$, and $(\mathbf{z}, s) \in \mathcal{Z}_k \times \mathcal{S}$.

We omit a formal proof of Theorem 4, given its immediacy in derivation. Essentially, the crux of the proof centers on showing that the EDF scheduling policy performs almost surely as well as any *non-causal* policy with perfect foresight – the so called oracle optimal policy. Technically, the proof relies on showing that any oracle optimal schedule can be inductively mapped to the EDF schedule without incurring an increase in the corresponding cost of firm supply, almost surely.

Appendix B
Intra-class Scheduling Policies

Formally, for a consumer i who purchases a bundle $\mathbf{a} = (a_1, \dots, a_N)$, we let $\boldsymbol{\lambda}_{k,i}(\mathbf{x}, \mathbf{a}) \in \mathbb{R}_+^N$ denote (element-wise in j) the amount of energy delivered (at period k) to consumer i so as to satisfy her demand a_j. We denote the intra-class scheduling policy by

$$\phi = \{(\boldsymbol{\lambda}_{0,i}, \dots, \boldsymbol{\lambda}_{N-1,i}) \mid i \in [0,1]\},$$

where $\boldsymbol{\lambda}_{k,i} : \mathbb{R}_+^N \times \mathcal{A} \to \mathbb{R}_+^N$ for all $i \in [0,1]$ and $k = 0, \dots, N-1$. Given an inter-class scheduling policy $\sigma \in \Sigma$, an intra-class scheduling policy ϕ is feasible if and only if it satisfies the following constraints.

1. The intra-class scheduling policy should not deliver any supply that is allocated to class j to consumers outside this class. That is, for each demand class $j = 1, \dots, N$, we have that $\lambda_{k,i}^j(\mathbf{x}, \mathbf{a}) = 0$ for every consumer i such that $a_j = 0$.

2. At every period k, no energy is delivered to demand class j with $j \le k$ by the feasibility of the inter-class policy $\sigma \in \Sigma$. That is, $\lambda_{k,i}^j(\mathbf{x}, \mathbf{a}) = 0$ for every $i \in [0,1]$, and every $1 \le j \le k \le N-1$.

3. The total supply allocated to demand class j at time period k must be fully utilized, i.e.

$$\int_{[0,1]} \lambda_{k,i}^j(\mathbf{x}, \mathbf{a})\, \eta(di) \;=\; \mu_k^j + \nu_k^j, \qquad \forall\; 0 \le k < j \le N-1,$$

where we use the Lebesgue integral (with respect to Lebesgue measure η defined on $[0,1]$), and μ_k^j and ν_k^j denote the amount of intermittent and firm supply allocated to demand class j at period k, according to the inter-class policy σ.

4. Each consumer's individual delivery commitments must be met:

$$\textstyle\sum_{t=1}^k a_t \;\le\; \omega_{k,i}^\pi(\mathbf{x}, \mathbf{a}) \;\le\; \textstyle\sum_{t=1}^N a_t, \qquad k = 1, \dots, N,$$

where the total energy delivered to consumer i by it's true deadline k_i is given by

$$\omega_{k_i,i}^\pi(\mathbf{x}, \mathbf{a}) = \sum_{t=0}^{k_i-1} \sum_{j=1}^{N} \lambda_{t,i}^j(\mathbf{x}, \mathbf{a}). \tag{16}$$

Notice that for any feasible inter-class scheduling policy $\sigma \in \Sigma$, it is always possible to ensure the satisfaction of the above constraint.

We denote the *feasible intra-class policy space* by $\Phi(\sigma)$, which is parameterized by a given inter-class policy $\sigma \in \Sigma$.

APPENDIX C
PROOF OF LEMMA 1

We first establish the simplified form of Q^* in Eq. (11). For notational simplicity, we denote the sequence of optimal inputs by $\mathbf{u}_k^* = \boldsymbol{\mu}_k^*(\mathbf{z}_k, s_k)$ and $\mathbf{v}_k^* = \boldsymbol{\nu}_k^*(\mathbf{z}_k, s_k)$ for $k = 0, \ldots, N-1$. Under the optimal scheduling policy, firm supply is deployed only as a last resort to ensure task satisfaction. It follows that

$$Q^*(\mathbf{x}) \;=\; c_0 \cdot \sum_{k=1}^{N} \mathbb{E}\left\{ v_{k-1}^{k,*} \right\}.$$

To establish the desired result, it suffices to show that $v_{k-1}^{k,*} = -\min\{0, \xi_k\}$. First, define the quantity

$$\delta_k \;=\; \sum_{j=0}^{k-1} s_j \;-\; \sum_{\ell=1}^{k}\sum_{j=0}^{\ell-1} u_j^{\ell,*}, \qquad \forall \;\; k = 1, \ldots, N,$$

which denotes the maximum amount of intermittent supply available to demand class $k+1$ across the first k time periods under a sequence of EDF allocations $\mathbf{u}_0^*, \ldots, \mathbf{u}_{k-1}^*$. Clearly, we have that $\delta_k \geq 0$ for all k, given feasibility of the allocations $\mathbf{u}_0^*, \ldots, \mathbf{u}_{k-1}^*$ under the intermittent supply availability constraints. One can readily show via an inductive argument that

$$\xi_k \;=\; \delta_k \;-\; v_{k-1}^{k,*}, \qquad \forall \;\; k = 1, \ldots, N.$$

Using this characterization of the residual process $\boldsymbol{\xi}$, we have that

$$\begin{aligned} \delta_k > 0 \;\;&\Longrightarrow\;\; v_{k-1}^{k,*} = 0 \;\;\Longrightarrow\;\; \min\{0, \xi_k\} = 0, \\ \delta_k = 0 \;\;&\Longrightarrow\;\; v_{k-1}^{k,*} \geq 0 \;\;\Longrightarrow\;\; \min\{0, \xi_k\} = -v_{k-1}^{k,*}, \end{aligned}$$

which yields the desired result that $\min\{0, \xi_k\} \;=\; -\, v_{k-1}^{k,*}$ and establishes the form of Q^* in Eq. (11).

We now establish *convexity* of the expected recourse cost $Q^*(\mathbf{x})$ directly. Let $\mathbf{x_1}, \mathbf{x_2} \in \mathbb{R}_+$ and denote the corresponding optimal scheduling policies by $\sigma_1^* \in \Sigma(\mathbf{x}_1)$ and $\sigma_2^* \in \Sigma(\mathbf{x}_2)$. Define the convex combination of demand bundles $\mathbf{x}_\lambda = \lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$, where $\lambda \in [0, 1]$. It follows that

$$\lambda Q^*(\mathbf{x}_1) + (1-\lambda) Q^*(\mathbf{x}_2) \;=\; \mathbb{E} \sum_{k=0}^{N-1} \mathbf{c}^\top (\lambda \mathbf{v}_k^{\sigma_1^*} + (1-\lambda)\mathbf{v}_k^{\sigma_2^*}).$$

And, it is not difficult to show that the convex combination of the constituent policies $\lambda\sigma_1^* + (1-\lambda)\sigma_2^*$ is admissible for the convex combination of demand bundles, i.e., $\lambda\sigma_1^* + (1-\lambda)\sigma_2^* \in \Sigma(\mathbf{x}_\lambda)$. Convexity of Q^* follows. *Differentiability* of $Q^*(\mathbf{x})$ over $(0, \infty)^N$ follows immediately from the proof of Theorem 2, in which we show that

$$\frac{\partial Q^*(\mathbf{x})}{\partial x_k} = \zeta_k(\mathbf{x}), \quad k = 1, \ldots, N,$$

where the function $\zeta_k(\mathbf{x})$, defined in (13), is bounded and continuous over $(0, \infty)^N$ for each $k = 1, \ldots, N$. ■

APPENDIX D
PROOF OF THEOREM 2

To simplify exposition, we employ the shorthand notation $(\cdot)^- = \min\{0, \cdot\}$. Fix a price bundle $\mathbf{p} \in \mathbb{R}_+^N$. We have previously shown in Theorem 1 that the EDF inter-class scheduling policy $\sigma^* \in \Sigma(\mathbf{x})$ is optimal for any demand bundle $\mathbf{x} \in \mathbb{R}_+^N$. Hence, it suffices to show that $\mathbf{p} = \boldsymbol{\zeta}(\mathbf{x})$ satisfies the first order condition for optimality (12). Taking the gradient of the supplier's expected profit with respect to $\mathbf{x}$ yields $\nabla_{\mathbf{x}} J(\mathbf{x}, \mathbf{p}, \sigma^*) = \mathbf{p} - \nabla_{\mathbf{x}} Q^*(\mathbf{x})$. It remains to show that

$$\frac{\partial Q^*(\mathbf{x})}{\partial x_k} \;=\; \zeta_k(\mathbf{x})$$

for $k = 1, \ldots, N$. Working the with the simplified form of Q^* established in Lemma 1, it follows readily that

$$\frac{\partial Q^*(\mathbf{x})}{\partial x_k} = c_0 \cdot \sum_{\ell=k}^{N} \frac{\partial}{\partial x_k} \mathbb{E}\left\{ \xi_\ell(\mathbf{x}, \mathbf{s})^- \right\}, \tag{17}$$

where we've truncated the summation from below at $\ell = k$, as $\xi_\ell(\mathbf{x}, \mathbf{s})$ is wholly independent of x_k for all $\ell < k$. We therefore restrict our attention to $\ell \geq k$ for the remainder of the proof. The next step of the proof relies on the ability to interchange the order of differentiation and expectation in (17). It is obvious from construction that $\xi_\ell(\mathbf{x}, \mathbf{s})^-$ is both a continuous function of $(\mathbf{x}, \mathbf{s})$ and piecewise affine in x_k (with a finite number of linear segments) for each $\mathbf{s}$. It follows that $\xi_\ell(\mathbf{x}, \mathbf{s})^-$ is differentiable almost everywhere in $x_k \in \mathbb{R}_+$ and satisfies

$$\left| \frac{\partial}{\partial x_k} \xi_\ell(\mathbf{x}, \mathbf{s})^- \right| \;\leq\; 1$$

almost everywhere. Then, for each $x_k \in \mathbb{R}_+$, we have that $\partial \xi_\ell(\mathbf{x}, \mathbf{s})^- / \partial x_k$ is integrable in x_k and by the dominated convergence theorem

$$\frac{\partial}{\partial x_k} \mathbb{E}\left\{ \xi_\ell(\mathbf{x}, \mathbf{s})^- \right\} \;=\; \mathbb{E}\left\{ \frac{\partial}{\partial x_k} \xi_\ell(\mathbf{x}, \mathbf{s})^- \right\}.$$

Finally, it is not difficult to see that

$$\frac{\partial}{\partial x_k} \xi_\ell(\mathbf{x}, \mathbf{s})^- \;=\; \begin{cases} \mathbf{1}_{\{\xi_k \leq 0\}}, & \ell = k, \\ \mathbf{1}_{\{\xi_\ell \leq 0\}} \cdot \displaystyle\prod_{t=k}^{\ell-1} \mathbf{1}_{\{\xi_t > 0\}}, & \ell > k. \end{cases}$$

And taking expectation, we have the desired result. ■

APPENDIX E
PROOF OF THEOREM 3

Let $\mathbf{x}$ be the aggregate demand of the other consumers (excluding i). Suppose that the supplier uses the EDF (inter-class scheduling) policy σ^*, and an arbitrary intra-class scheduling policy $\phi \in \Phi(\sigma^*)$. We let $\pi^* = (\sigma^*, \phi)$. We consider a consumer i of some type $\theta_i = (k, R, q)$ such that $c_0 \leq R$. We will show that the consumer, who faces an arbitrary aggregate demand bundle requested by other consumers $\mathbf{x}$, would like to take the truth-telling action defined in Eq. (**??**). For the rest of this proof, we will use $\mathbf{p} = \{p_k\}_{k=1}^N$ to denote the price bundle induced by the pricing scheme $\boldsymbol{\zeta}$ (cf. its definition in Eq. (13)), at the aggregate demand $\mathbf{x}$.

If consumer i is truth-telling, she will request a quantity q before deadline k, and receive an expected payoff of

$$\begin{aligned} &V_i^{\pi^*}(\theta_i, \varphi_i^*, \mathbf{x}, \boldsymbol{\zeta}) \\ &= U(q) - qp_k \\ &= qR - qc_0\Big[\mathbb{P}(\xi_k \le 0) \\ &\qquad + \sum\nolimits_{t=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{t-1} > 0, \xi_t \le 0)\Big]. \end{aligned} \tag{18}$$

Since the optimal price schedule is nonincreasing in deadline, and demand is guaranteed to be met before the requested deadline, the consumer has no incentive to request a positive amount of electricity at some period t that is earlier than k. We can therefore assume that consumer i takes an action $\mathbf{a}' = \varphi_i'(\theta_i)$ such that

$$a_t' = 0, \qquad t = 0, \ldots, k-1.$$

Since the consumer cannot increase its expected payoff (compared to being truth-telling) by reporting $a_k' \ge q$, we focus on the case where $a_k' < q$. We first write the consumer's expected payoff (achieved by the action $\mathbf{a}'$) as

$$V_i^{\pi^*}(\theta_i, \varphi_i', \mathbf{x}, \boldsymbol{\zeta}) = \mathbb{E}\left\{U_\theta\left(\omega_{k,i}^{\pi^*}(\mathbf{x}, \mathbf{a}')\right)\right\} - \sum_{t=k}^{N} p_t a_t'.$$

Showing that $V_i^{\pi^*}(\theta_i, \varphi_i', \mathbf{x}, \boldsymbol{\zeta})$ is no more than the expected payoff in (18) is equivalent to

$$\begin{aligned} &Rq - (q - a_k')c_0\Big[\mathbb{P}(\xi_k \le 0) \\ &\qquad + \sum\nolimits_{t=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{t-1} > 0, \xi_t \le 0)\Big] \\ &\ge \mathbb{E}\left\{U_\theta\Big(\omega_{k,i}^{\pi^*}(\mathbf{x}, \mathbf{a}')\Big)\right\} - \sum\nolimits_{t=k+1}^{N} p_t a_t'. \end{aligned} \tag{19}$$

We will derive an upper bound on the right hand side of (19), and show that this upper bound cannot exceed the left hand side of (19). For notational convenience, we define

$$\eta_t = \{\xi_k > 0, \ldots, \xi_{t-1} > 0, \xi_t \le 0\}$$

for all $t = k+1, \ldots, N-1$ and

$$\eta_k = \{\xi_k \le 0\}, \qquad \eta_N = \{\xi_k > 0, \ldots, \xi_{N-1} > 0\}.$$

Note that these $N - k + 1$ events are mutually disjoint. Further, it is straightforward to see that $\xi_k \le 0$ implies that $\omega_{k,i}^{\pi}(\mathbf{x}, \mathbf{a}') = a_k'$. While, on the other hand, $\xi_k > 0$ implies that one of the (mutually disjoint) events $\{\eta_t\}_{t=k+1}^{N}$ must occur, i.e.,

$$1 = \mathbb{P}\left(\bigcup\nolimits_{t=k}^{N} \eta_t\right) = \sum_{t=k}^{N} \mathbb{P}(\eta_t). \tag{20}$$

Under the event η_t, the amount of energy delivered by the EDF scheduling policy before her true deadline k, $\omega_{k,i}^{\pi^*}(\mathbf{x}, \mathbf{a}')$, cannot exceed $\sum_{\tau=k}^{t} a_\tau'$, for $t = k, \ldots, N$. It follows from Assumption 1 that

$$Rq - U_\theta(x) \ge R\,(q - x)^+, \qquad \forall\, x \ge 0,$$

where $(\cdot)^+ = \max\{\cdot, 0\}$. We then have

$$\begin{aligned} &Rq - \mathbb{E}\left\{U_\theta\left(\omega_{k,i}^{\pi^*}(\mathbf{x}, \mathbf{a}')\right)\right\} \\ &\ge R\sum\nolimits_{t=k}^{N} \mathbb{P}(\eta_t) \cdot \Big(q - \sum\nolimits_{\tau=k}^{t} a_\tau'\Big)^+. \end{aligned} \tag{21}$$

We now argue that the right hand side of (21) is minimized at some vector $\tilde{\mathbf{a}}$ such that $\sum_{t=k}^{N} \tilde{a}_t \le q$. To see this, suppose that $\sum_{t=k}^{N} a_t' > q$. Let T be the smallest t such that $\sum_{m=k}^{t} a_m' > q$. We define an alternative vector $\tilde{\mathbf{a}}$

$$\tilde{a}_t = a_t', \qquad t \le T-1, \qquad \tilde{a}_T = q - \sum_{m=k}^{T-1} a_m', \tag{22}$$

and $\tilde{a}_t = 0$, for every $t > T$. We have $\sum_{t=k}^{N} \tilde{a}_t = q$, and

$$\begin{aligned} &\sum\nolimits_{t=k}^{N} \mathbb{P}(\eta_t) \cdot \Big(q - \sum\nolimits_{\tau=k}^{t} a_\tau'\Big)^+ \\ &= \sum\nolimits_{t=k}^{N} \mathbb{P}(\eta_t) \cdot \Big(q - \sum\nolimits_{\tau=k}^{t} \tilde{a}_\tau\Big)^+. \end{aligned}$$

Note that if $\sum_{t=k}^{N} a_t' \le q$, then $\tilde{\mathbf{a}} = \mathbf{a}'$. To validate (19), it suffices to show that for any vector $\tilde{\mathbf{a}}$ defined in (22),

$$\begin{aligned} &R\sum\nolimits_{t=k}^{N} \mathbb{P}(\eta_t) \cdot \Big(q - \sum\nolimits_{\tau=k}^{t} \tilde{a}_\tau\Big) \\ &\ge -\sum\nolimits_{t=k+1}^{N} p_t \tilde{a}_t + (q - \tilde{a}_k)c_0\Big[\mathbb{P}(\xi_k \le 0) \\ &\qquad + \sum\nolimits_{t=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{t-1} > 0, \xi_t \le 0)\Big], \end{aligned} \tag{23}$$

where the left hand side is a lower bound on the loss of expected utility due to the non-truthful action $\tilde{\mathbf{a}}$ (cf. the inequality in (21)), and the right hand side is the difference between the payment under the truth-telling action and the action $\tilde{\mathbf{a}}$. We will prove the following inequality that is equivalent to (23)

$$\begin{aligned} &R\sum\nolimits_{t=k}^{N} \mathbb{P}(\eta_t) \cdot \Big(q - \sum\nolimits_{\tau=k}^{t} \tilde{a}_\tau\Big) \\ &= R(q - \tilde{a}_k) - R\sum\nolimits_{t=k+1}^{N} \Big(\mathbb{P}(\eta_t) \cdot \sum\nolimits_{\tau=k+1}^{t} \tilde{a}_\tau\Big) \\ &= R(q - \tilde{a}_k) - R\sum\nolimits_{t=k+1}^{N} \Big(\tilde{a}_t \cdot \sum\nolimits_{\tau=m}^{N} \mathbb{P}(\eta_m)\Big) \\ &\ge (q - \tilde{a}_k)c_0\Big[\mathbb{P}(\xi_k \le 0) \\ &+ \sum_{t=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{t-1} > 0, \xi_t \le 0)\Big] - \sum\nolimits_{t=k+1}^{N} p_t \tilde{a}_t, \end{aligned} \tag{24}$$

where the first equality follows from (20), and the second equality is obtained by rearranging terms.

It follows from the characterization of supplier marginal cost in (13) that for $t = k+1, \ldots, N$,

$$\begin{aligned} \tilde{a}_t p_t &= \tilde{a}_t c_0 \sum_{m=t}^{N} \mathbb{P}(\xi_t > 0, \ldots, \xi_{m-1} > 0, \xi_m \le 0) \\ &\ge \tilde{a}_t c_0 \sum_{m=t}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{m-1} > 0, \xi_m \le 0). \end{aligned} \tag{25}$$

We then have, for $t = k+1, \ldots, N$,

$$\begin{aligned}
&R\tilde{a}_t \sum_{m=t}^{N} \mathbb{P}(\eta_m) - \tilde{a}_t p_t \\
&= R\tilde{a}_t \left(1 - \mathbb{P}(\xi_k \leq 0) - \sum_{m=k+1}^{t-1} \mathbb{P}(\eta_m)\right) - \tilde{a}_t p_t \\
&\leq R\tilde{a}_t \left(1 - \mathbb{P}(\xi_k \leq 0) - \sum_{m=k+1}^{t-1} \mathbb{P}(\eta_m)\right) \\
&\qquad -\tilde{a}_t c_0 \sum_{m=t}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{m-1} > 0, \xi_m \leq 0) \\
&\leq R\tilde{a}_t \left(1 - \mathbb{P}(\xi_k \leq 0)\right) \\
&\qquad -\tilde{a}_t c_0 \sum_{m=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{m-1} > 0, \xi_m \leq 0) \\
&\leq R\tilde{a}_t - \tilde{a}_t c_0 \Big(\mathbb{P}(\xi_k \leq 0) \\
&\qquad + \sum_{m=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{m-1} > 0, \xi_m \leq 0)\Big).
\end{aligned} \tag{26}$$

Here, the first inequality follows from (25); the second inequality is true, because $R \geq c_0$ and $\eta_m = \{\xi_k > 0, \ldots, \xi_{m-1} > 0, \xi_m \leq 0\}$, for $m = k+1, \ldots, t-1$; the last inequality follows from $R \geq c_0$. For any vector $\tilde{\mathbf{a}}$ with $\sum_{t=k}^{N} \tilde{a}_t \leq q$, from (26) we have

$$\begin{aligned}
&\sum_{t=k+1}^{N} \left(-p_t \tilde{a}_t + R\tilde{a}_t \sum_{m=t}^{N} \mathbb{P}(\eta_m)\right) \\
&\leq \sum_{t=k+1}^{N} \tilde{a}_t \Big[R - c_0 \Big(\mathbb{P}(\xi_k \leq 0) \\
&\qquad + \sum_{m=k+1}^{N} \mathbb{P}(\xi_k > 0, \ldots, \xi_{m-1} > 0, \xi_m \leq 0)\Big)\Big].
\end{aligned} \tag{27}$$

Since $R \geq c_0$, the right hand side of (27) is nondecreasing in $\sum_{t=k+1}^{N} \tilde{a}_t$. The desired result in (24) follows from the fact $\sum_{t=k+1}^{N} \tilde{a}_t \leq q - \tilde{a}_k$. Since the preceding analysis holds for any action $\mathbf{a}'$ and any aggregate demand bundle $\mathbf{x}$, we conclude that it is a dominant strategy for consumer i to be truth-telling, i.e., the pricing scheme (13) is incentive compatible, in the sense of Definition 6. ■

APPENDIX F
PROOF OF COROLLARY 1

We first show that $(\mathbf{x}^*, \boldsymbol{\zeta}(\mathbf{x}^*))$ is a market equilibrium (in the sense of Definition 7), and then argue that it is the unique market equilibrium that maximizes the social welfare. Under the mechanism $(\sigma^*, \phi, \boldsymbol{\zeta})$, it follows from Assumption 3 and Theorem 3 that it is a dominant strategy for every consumer to be truth-telling, and therefore the aggregate demand is $\mathbf{x}^*$. Given the price bundle $\boldsymbol{\zeta}(\mathbf{x}^*)$, the aggregate demand bundle $\mathbf{x}^*$ together with the EDF scheduling policy maximizes the supplier's expected profit (cf. Theorem 2). Hence, the pair, $(\mathbf{x}^*, \boldsymbol{\zeta}(\mathbf{x}^*))$, constitutes a market equilibrium.

Let $(\mathbf{x}, \mathbf{p})$ be a market equilibrium. The second condition of Definition 7 requires that the given the price bundle $\mathbf{p}$, the quantity $\mathbf{x}$ maximizes the (price-taking) supplier's expected profit. We note that this implies that $\mathbf{p}$ is the supplier's marginal cost to supply $\mathbf{x}$. Since the supplier marginal cost never exceeds c_0, at any price bundle that represents supplier marginal cost, a truth-telling consumer would request her maximum demand by her true deadline, and therefore the aggregate demand of a truth-telling consumer population is always $\mathbf{x}^*$. The uniqueness of the truth-telling aggregate demand implies the uniqueness of a market equilibrium.

It is straightforward to see that social welfare is maximized at this market equilibrium, because under Assumption 3, it is socially optimal to fully serve the aggregate demand $\mathbf{x}^*$, and further, the EDF scheduling policy σ^* minimizes the expected cost of servicing $\mathbf{x}^*$. ■