Optimal Influence Under Observational Learning

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Optimal Influence Strategies

General question of finding **Optimal Influence Strategies**:

- Optimal selection of influential agents within a society.

Influential Agents are called those:

- whose early adoption of a certain behavior/action can trigger its maximum spread in the society.

Literature in economics, marketing, sociology, comp sci, physics...

Motivating Example

A firm wants to spread a product in a new market:

- Consumers' choices are influenced by those of "neighbors"
  - *If they* **observe** *a product that performs better, they buy it.*
- The firm is **uncertain** about the **performance** of the product
  - *Random factors (weather etc.),*
  - *Unexpected problems (operating system crashing etc.).*
- The firm disseminates information by **targeting agents**
  - *Convinces some of them to choose the product once.*

WHO SHOULD THE FIRM TARGET?

The General Question

More General Question:

Propagate a technology/product/idea in a society where agents are influenced by their neighbors:

$\rightarrow$ *A firm promotes a new product*.
$\rightarrow$ *A government promotes the adoption of a new technology.*
$\rightarrow$ ...
$\rightarrow$ General: *A principal seeks to maximize the diffusion of an action*

How? By targeting optimally a subset of the population.

The Setting

- Agents choose repeatedly between two products A and B.
- Observe qualities of products used themselves and neighbors.
  - Learn which is better when observe both.
- Firm can target a subset of the population.
- Firm seeks to maximize diffusion of a product B in a market.
  - Lifetime discounted sum of consumed products B.

Aspects of Interest

Identify the optimal targeting strategy of a firm

- ex–ante uncertain about the relative quality of its product,
  - which does not change across periods.
  - learn it only after the product has been circulated.
- agents perfectly learn the quality of a product
  - as soon as they have observed someone using it.
  - compare the results with those without learning.

The Mechanism

Product is of higher quality than its alternative:

- Will get diffused to the whole population,
- The firm would like diffusion to occur as fast as possible.

Product is of lower quality than its alternative:

- Will survive only for a finite number of periods,
- Until every agent has observed the alternative at least once.
- Protect the early consumers from observing the alternative.

The Agents

- Finite set of agents, located on a connected social network.
- Geodesic distance, $d(i,j)$, between two agents is the length of the shortest path between them.
- Geodesic distance, $d(T,j)$, between a set of agents T and an agent outside the set, $j \in T^C$, is defined as the minimum among all the distances between some agent $i \in T$ and j.
- Consume each period one unit of product A or B.
- Observe qualities of products they consume.
- Observe qualities of products their neighbors consume.
- Learn relative quality of products when they observe both.

Geodesic Distance between agents

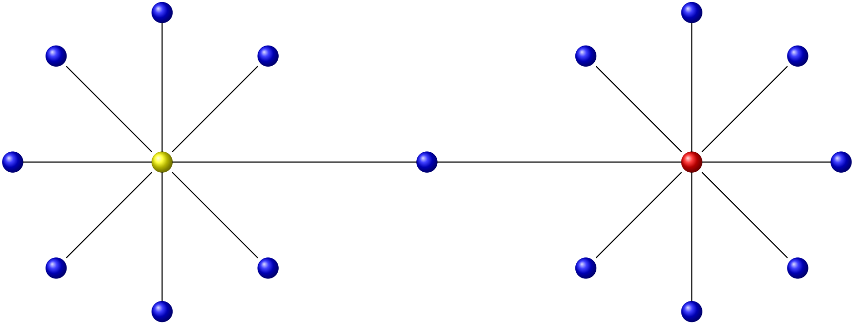


Figure: Geodesic Distance between yellow and red is two

Geodesic Distance between agent and set of agents

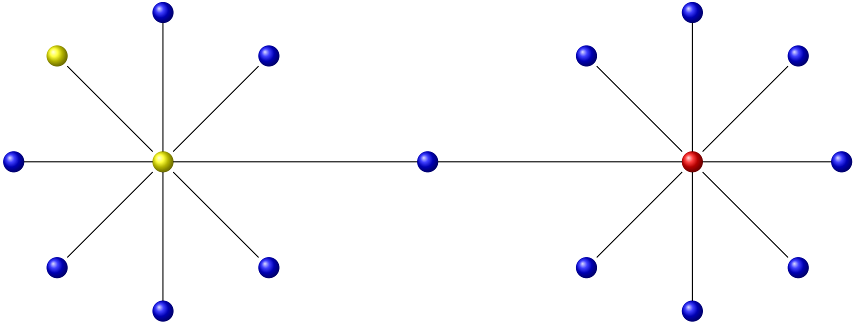


Figure: Geodesic Distance between yellow set and red agent is two

The Firm

- Does not know the relative quality of B compared to A.
- Knows that it is of higher quality w.p. $\underline{p \in [0,1]}$.
- Can target initially t agents to consume B once.
- Discounts future sales with $\underline{\delta \in [0,1]}$.
- Firm maximizes discounted sum of products B consumed.
- For a targeting choice T, this is

$$\Pi(T) := E_{q_B}\left[\sum_{\tau=1}^{+\infty} \delta^{\tau-1} S_\tau(T)\right] =$$

$$= p \sum_{\tau=1}^{+\infty} \delta^{\tau-1} S_\tau(T|q_B = H) + (1-p) \sum_{\tau=1}^{+\infty} \delta^{\tau-1} S_\tau(T|q_B = L)$$

Targeting One Agent - General Case

Definition

For $\delta \in (0,1)$, decay centrality of agent $i \in N$ is $\sum_{j \neq i} \delta^{d(i,j)}$.

- How close an agent is to other agents,
- Very distant agents are weighted less than closer ones.
- Agents with high decay centrality combine:
  - higher number of neighbors,
  - small geodesic distance from isolated agents.

Theorem

For all $(\delta, p) \in (0,1)^2$, the optimal strategy for the firm is to target the agent $i \in N$ who maximizes decay centrality.

Targeting One Agent - $\delta = 0$ and $\delta = 1$

Corollary

If $\delta = 0$, then the optimal strategy for the firm is to target the agent $i \in N$ with the maximum number of neighbors.

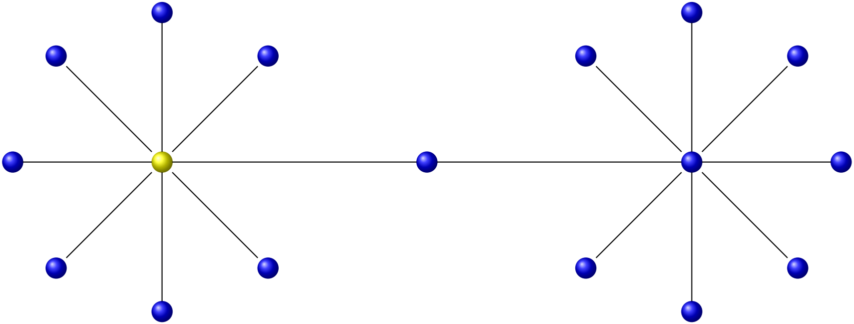
Definition

The farness of an agent $i \in N$ is the sum of the geodesic distances from each other agent in the network, i.e. $\sum_{j \neq i} d(i, j)$.

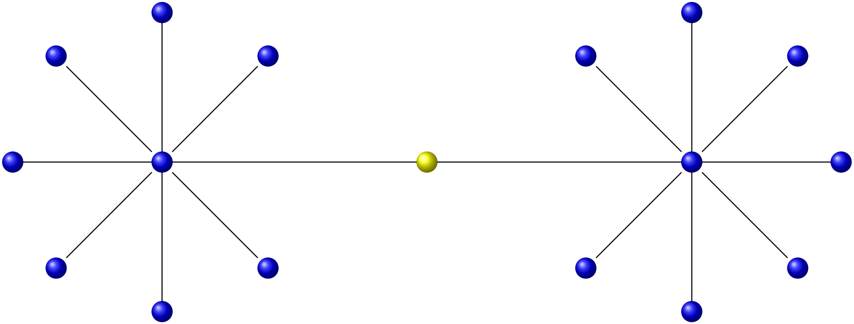
Proposition

If $\delta = 1$, then the optimal strategy for the firm is to target the agent with the minimum farness (maximum closeness).*

Agent with maximum degree



Agent with minimum farness



Targeting Several Agents - General Case

Definition

Consider a decay parameter $\delta \in (0,1)$. The *group decay centrality* of set T is defined as $\sum_{j \in T^C} \delta^{d(T,j)}$.

Theorem

For $(\delta, p) \in (0,1)^2$, among all subsets of N with t agents the optimal strategy is to target the set that maximizes:

$$p \sum_{j \in T^C} \delta^{d(T,j)} - (1-p) \sum_{i \in T} \delta^{d(T^C,i)}$$

- Maximize decay centrality of the set.
- Minimize decay centrality of its complement.
- Not very easy to picture it in general networks.

Targeting Several Agents - $p = 0$ and $p = 1$

Corollary

For $p = 1$, among all subsets of N with t agents the optimal strategy is to target the set:

- *with the maximum group degree, if $\delta = 0$.*
- *with the maximum closeness, if $\delta = 1$.*
- *with the maximum decay centrality, if $\delta \in (0, 1)$.*

Corollary

For $p = 0$, among all subsets of N with t agents the optimal strategy is to target the set:

- *whose complement has the min group degree, if $\delta = 0$.*
- *whose complement has the min closeness, if $\delta = 1$.*
- *whose complement has the min decay centrality, if $\delta \in (0, 1)$.*

The Circle

Proposition

For a given number of targeted groups it is optimal to:

1. *Concentrate most of the targeted agents in one group,*
2. *Spread the rest of the targets around the network,*
3. *Such that non–targeted are split as equally as possible.*

The Circle - II

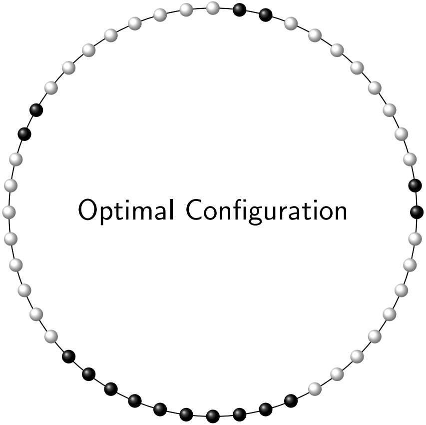


Figure: The optimal targeting strategy if the firm targets four groups.

Intuition behind this result

1. First condition (in case product is of low quality):
    - Min decay centrality of the complement of the targeted set.
    - Max number of periods until all targets observe the alternative.

2. Second and third (in case product is of high quality):
    - Maximize the decay centrality of the targeted set.
    - Make the product visible to everyone quickly.

The Circle - How Many Groups - I

Proposition

Assume that the firm can only target either one or two groups. Then for any $(p, \delta) \in (0,1)^2$ *it is optimal to target two groups if and only if* $p \geq \frac{(1-\delta)\left(1-\delta^{\frac{t}{2}-1}\right)}{(1-\delta)\left(1-\delta^{\frac{t}{2}-1}\right)+\left(1-\delta^{\frac{s}{4}}\right)^2}$.

where t is number of targeted agents and $s = n - t$.

The Circle - How Many Groups - II

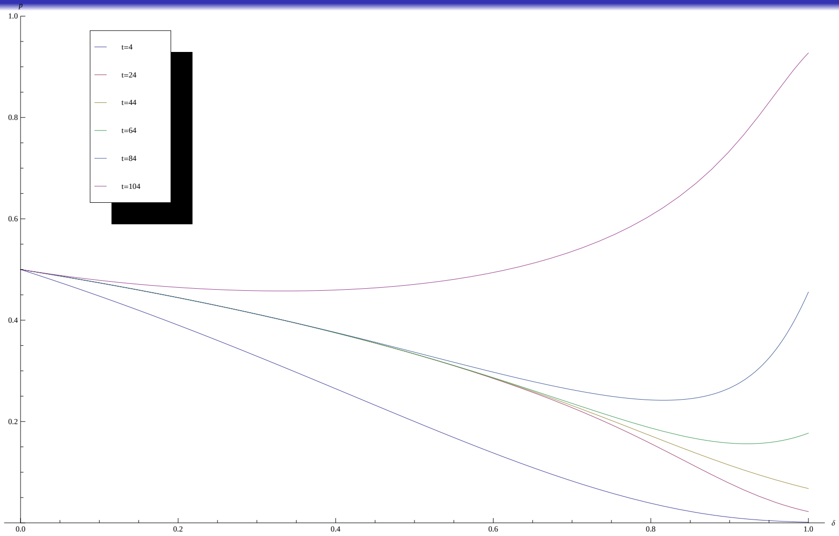


Figure: Choice between either one or two groups for different values of t.

The Circle - How Many Groups - III

Proposition

Assume that the firm can target any number of groups, g, then we can find an $f(\delta, t, g)$ such that the optimal strategy for the firm is to target:

- *One group if $p \leq \min_{g \geq 1} f(\delta, t, g)$*
- *The maximum number of groups if $p \geq \max_{g \geq 1} f(\delta, t, g)$*

The Circle - How Many Groups - IV

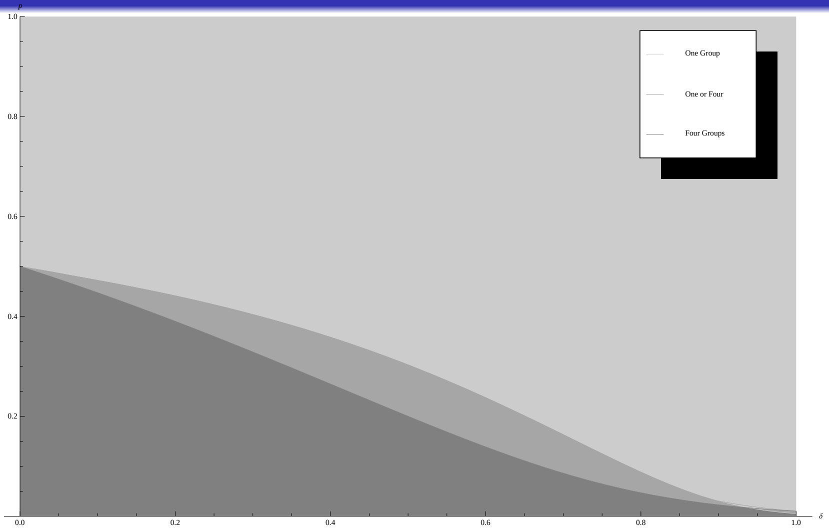


Figure: (i) One group in the dark gray area, (ii) Max number of groups in the light gray area, (iii) Compare in the intermediate area.

Conclusion

- Analyzed the optimal strategy of a firm who seeks to maximize the diffusion of a product of uncertain quality in a society.
- Crucial parameter is the agents' decay centrality
  - Depends vastly on the exact topology of the network.
  - Identifying agents who maximize decay centrality in a network has been answered only in very simple network structures.
  - Even harder if we pass to group decay centrality.
  - Systematic study of this problem can shed light to a number of environments where decay centrality seems to be crucial.