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Non-Binary Protograph-Based LDPC Codes for 2D ISI Magnetic Recording Channels

Pingping CHEN[1,2], Kui Cai[2], *Senior Member, IEEE*, Lingjun Kong[3], Zaichun Chen[2], and Mu Zhang[2]
[1] Fuzhou University, Fujian, China, [2] Singapore University of Technology and Design, Singapore
[3] College of Telecommunication & Inform. Engineering, Nanjing University of Posts & Telecommunications, Nanjing, China

Two dimensional inter-symbol-interference (2D-ISI), consisting of ISI in the down track direction and inter-track interference (ITI) along the cross-track direction, is a major factor that severely degrades the performance of ultra-high density magnetic recording systems. Due to its superior performance and the protograph structure which facilitates high-speed encoding and decoding, the protograph codes have shown a high potential to tackle 2D-ISI. However, so far no work has been reported on the design of non-binary protograph-based low-density parity-check (NP-LDPC) codes for 2D-ISI channels. In this work, we first propose a modified protograph extrinsic information transfer (EXIT) analysis. In conjunction with the asymptotic ensemble weight distribution (AWE) analysis, they both serve as theoretical tools to analyze the performance of NP-LDPC codes designed for the 2D-ISI channels. By further applying a fast search approach, we construct two types of NP-LDPC codes for 2D-ISI channels. Both theoretical analyses and simulated results show that the proposed codes outperform the existing binary LDPC codes that optimized for 2D-ISI channels as well as the non-binary quasi-cyclic (NQC) LDPC code, at both low and high SNR regions.

***Index Terms*—Magnetic recording channels, two dimensional inter-symbol-interference (2D-ISI), protograph low-density parity-check (LDPC) codes, extrinsic information transfer (EXIT) analysis**

I. Introduction

THe conventional perpendicular magnetic recording system is approaching its area density (AD) limit [1], the bit-patterned magnetic recording (BPMR), heat (or microwave) assisted magnetic recording, and shingled writing/two-dimensional magnetic recording (TDMR) have been proposed in recent years to extend the storage density beyond 1 Tb/in^2 [2], [3], [4]. Correspondingly, both the bit-length and track-pitch of the magnetic storage media are reduced significantly, leading to severe inter-symbol-interference (ISI) in the down track direction, and inter-track interference (ITI) along the cross-track direction. ISI and ITI together give rise to a two-dimensional (2D)-ISI for ultra-high density magnetic recording systems. Advanced channel coding and signal processing techniques play a critical role to effectively mitigate the 2D-ISI and enable a higher AD for the magnetic recording systems [5]-[9].

Due to their superior capacity-approaching performance, the low-density parity-check (LDPC) codes have been widely applied in the read channel of magnetic recording systems. Many works have been done to investigate the binary LDPC codes and their error floor performance in the one-dimensional (1D)-ISI magnetic recording channels which are also modeled as the partial response (PR) channels [10], [11], [12]. For ultra-high density magnetic recording systems with 2D-ISI, the binary irregular LDPC codes have been optimized by using extrinsic information transfer (EXIT) curve fitting [13]. Since non-binary LDPC (NB-LDPC) codes are found to outperform their binary counterparts for additive white Gaussian noise (AWGN) channels, the authors in [14] investigated the performance of non-binary quasi-cyclic (NQC) LDPC codes based on progressive-edge-growth (PEG) algorithm for the PR channels. On the other hand, the protograph-based LDPC codes, constructed from the small base matrices or protographs, have shown excellent performance with simple hardware implementation for AWGN channels and PR channels [15], [10]. Recently, several types of binary protograph-based LDPC codes have been designed for 2D-ISI channels [16], [17], [18]. However, so far no work has been reported on the design of non-binary protograph-based LDPC (NP-LDPC) codes for 2D-ISI channels.

In this work, we present novel NP-LDPC codes designed for 2D-ISI magnetic recording channels, based on a modified protograph extrinsic information transfer (EXIT) analysis, together with the asymptotic ensemble weight distribution (AWE) analysis. By further applying a fast search approach, we construct two types of NP-LDPC codes. The EXIT-chart analysis, the AWE analysis, and the error rate simulations have demonstrated the superior performance of the proposed codes over the existing binary LDPC codes and the NQC LDPC code for 2D-ISI channels.

The paper is organized as follows. In Section II, we introduce the system model of a non-binary LDPC coded system with 2D-ISI. In Section III, we propose a modified protograph EXIT analysis, together with the AWE analysis, for analyzing the codes' performance for 2D-ISI channels. Two types of NP-LDPC codes are constructed for 2D-ISI magnetic recording channels in Section IV, and their performances are evaluated in Section V. Finally, Section VI concludes the paper.

II. System Model

In a magnetic recording channel with 2D-ISI [5], each bit suffers interferences from both the down-track and the cross-track directions. The channel response before equalization can be represented by a $L_M \times L_N$ matrix H, with L_M and L_N being the interference lengths in the down-track and cross-track directions, respectively. In this paper, we consider the channel response matrix corresponding to a BPMR system with a recording density of 4Tb/in^2 [13]. The corresponding block diagram of a non-binary LDPC coded system with 2D-ISI is illustrated by Fig. 1. The non-binary information vector with each element being selected from $GF(q), q = 2^p$, is first encoded to a non-binary vector u of length K using a non-binary LDPC code with a code rate of R. Considering an element $\alpha \in GF(q)$, it has an associated polynomial $f_\alpha(x)$ with binary coefficients given by

$$f_\alpha(x) = \sum_{k=1}^{p} \alpha(k) \cdot x^{k-1}, \alpha(k) \in \{0, 1\}. \quad (1)$$

The code vector u is then mapped into a binary vector by converting each symbol in u into a binary subvector of p bits. Let u_l denote the lth symbol with a value α in u and let $u_l(j)$ denote the mapped jth bit within u_l. We thus have $[u_l(1), u_l(2), ..., u_l(p)] = [\alpha(1), \alpha(2), ..., \alpha(p)]$, with $l = 1, 2, ..., K$. The whole binary vector of length Kp is converted into a 2D data block **x** with M rows and N columns by a BPSK modulator, i.e., $x(i, j)$ equals 1 and -1 for a binary bit equals 0 and 1, respectively, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, N, MN = Kp$. Then, the received signal at position (i, j) can be expressed by

$$r_{i,j} = \sum_{s=0}^{L_M-1} \sum_{t=0}^{L_N-1} h_{(s,t)} x_{(i-s+d_M, j-t+d_N)} + n_{(i,j)}, \quad (2)$$

where d_M and d_N are non-causality offsets, $h_{(s,t)}$ is a element of the H matrix. and $n_{(i,j)}$ is the AWGN noise with zero mean and variance σ_n^2. Generally, we set the offsets to be $d_M = L_M - 1/2$ and $d_N = L_N - 1/2$. We set a boundary of -1S surrounding the 2D data block for initializing the channel detector, i.e., $x(i, j) = -1$ for $(i, j) \notin \{(i, j)|1 \le i \le M, 1 \le j \le N\}$. Moreover, in the coded 2D-ISI systems, we define the bit SNR per bit as

$$\Lambda_b = 10\log10\left(\frac{\sum_{s,t} |h_{(s,t)}|^2}{2\sigma_n^2 \cdot R}\right) \quad (3)$$

This paper uses the fast Fourier transform based non-binary sum-product algorithm (FFT-SPA) LDPC decoder and the symbol-based Bahl-Cocke-Jelinek-Raviv (BCJR) detector [20] to iteratively recover the original messages in the 2D-ISI channel. The extrinsic messages from non-binary LDPC decoder are transformed into binary extrinsic messages before feeding back to the BCJR detector. Let v_i denote the variable node associated with the u_i. The binary *a priori* messages for the detector, corresponding to the bit $u_i(j)$, denoted by $L_a(u_i(j))$ are given in (4), where $s(u_i)$ denotes the check node set associated with v_i, and $p_{c,u_i}(u_i = \alpha)$ is the extrinsic message from the check node c to v_i, $c \in s(u_i)$. Here, $p_e(u_i(j))$ is the extrinsic message of $u_i(j)$ obtained from the BCJR detector in the previous iteration.

III. Analysis of Non-binary Protograph-based LDPC Codes for 2D-ISI Channels

In a non-binary LDPC coded 2D-ISI system, the non-binary code and the 2D-ISI channel are considered as the outer and inner codes of a concatenated coding system. Fig. 2 shows the block schematic of a joint graph of the NP-LDPC protograph with the 2D-ISI channel trellis. Considering a protograph consisting of a set $V = \{v_1, v_2, ..., v_{n_v}\}$ of variable nodes and a set $C = \{c_1, c_2, ..., c_{n_c}\}$ of check nodes, we propose a modified non-binary protogpraph EXIT analysis and further apply the AWE analysis to evaluate the performance of NP-LDPC coded 2D-ISI channel in the waterfall region and the error floor region respectively.

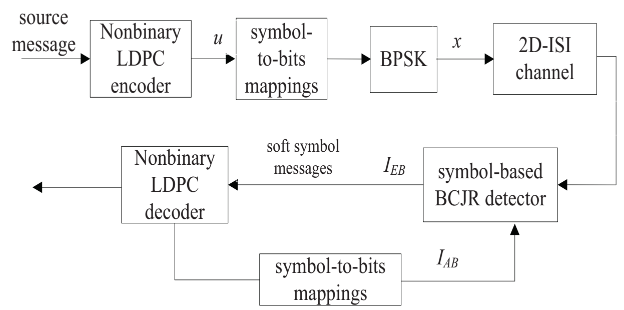

Fig. 1. Block diagram of a non-binary channel coded system with 2D-ISI.

A. Modified Protograph EXIT Analysis

For LDPC codes over GF(q), the $(q-1)$-dimensional log-likelihood-ratio (LLR) message, denoted by L, is approximated by the Gaussian distribution with a mean vector m and a covariance matrix $\sum$, with $m_i = \sigma^2/2, i = 1, ..., q-1$, and $\sum_{ij} = \sigma^2$ if $i = j$ and $\sum_{ij} = \sigma^2/2$ otherwise [19]. The mutual information between the transmitted symbol u and the LLR message L sent from check-to-variable nodes (c-to-v), can be computed as [19]

$$I(u; L) = 1 - \mathbb{E}\left[\log_q\left(1 + \sum_{i=1}^{q-1} e^{-L_i}|u = 0\right)\right], \quad (5)$$

where L_i is the ith entry in L. We can use the $J(\sigma)$ function to approximate (5), with σ being the parameter of the multivariate Gaussian distribution. Moreover, since the variable-to-check nodes (v-to-c) LLR messages may not be well approximated by a Gaussian random variable, the mutual information between v-to-c messages and transmitted symbols is computed by a Monte-Carlo method based function $J_R(\sigma_A, \sigma)$ [19], with σ_A being the variance of *a priori* LLR message of the decoder.

An iterative decoding threshold of a protograph is the minimum channel quality that ensures successful decoding for infinite-length LDPC codes built from the protograph. To compute the iterative threshold of the joint protograph shown in Fig. 2, we propose a scheme that combines the EXIT analysis for the BCJR detector with that for the non-binary protograph, so as to evaluate the performance of NP-LDPC coded 2D-ISI channel with turbo equalization. For convenience, we define four types of mutual information as follows.

- I_{AB} and I_{EB} denote *a priori* and extrinsic mutual information of the BCJR detector.
- $I_{EV}(i, j)$ denotes extrinsic mutual information between v_j-to-c_i LLR message and the corresponding coded symbol v_j.

$$L_a(u_i(j)) = \log \frac{\sum\limits_{\alpha\in GF(q):\alpha(i)=0}\ \sum\limits_{c\in s(u_i)} p_{c,u_i}(u_i=\alpha) \prod\limits_{k=1,k\neq j}^{p} p_e(u_i(k)=\alpha(k))}{\sum\limits_{\alpha\in GF(q):\alpha(i)=1}\ \sum\limits_{c\in s(u_i)} p_{c,u_i}(u_i=\alpha) \prod\limits_{k=1,k\neq j}^{p} p_e(u_i(k)=\alpha(k))}, \tag{4}$$

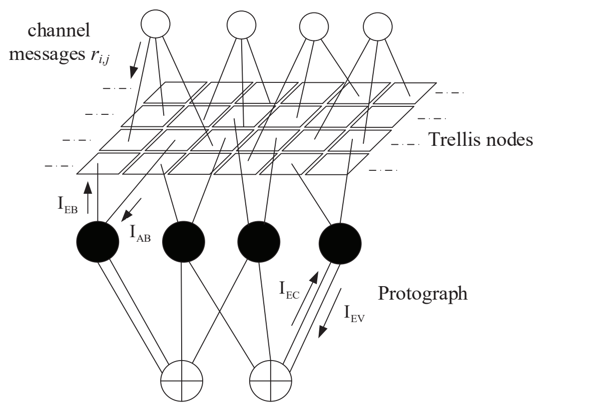

Fig. 2. A joint graph of the NP-LDPC protograph with the 2D-ISI channel trellis.

- $I_{EC}(i,j)$ denotes extrinsic mutual information between c_i-to-v_j LLR message and the corresponding coded symbol v_j.
- $I_{AP}(j)$ denotes *a posteriori* mutual information between *a posteriori* LLR message and the corresponding coded symbol v_j.

The modified EXIT for the joint protograph is then summarized as follows:

1) Mutual information of the BCJR detector. For the given bit SNR Λ_b and the *a priori* mutual information I_{AB}, we can obtain extrinsic LLR messages from the BCJR detector by using Monte Carlo simulations. We measure the variance of the detector output LLR σ_{EB}^2 and then compute the extrinsic mutual information I_{EB} by using J function.

2) Mutual information of variable nodes. The mutual information $I_{EV}(i,j)$ from v_j to c_i depends on the output LLR of the detector and the incoming messages from the check nodes. Since v_i-to-c_j LLR messages $I_{EV}(i,j)$ are not well approximated by a Gaussian distribution for $q \geq 2$, we compute it by using the J_R function. We also compute the *a priori* mutual information I_{AB} output from the decoder to the detector.

3) Mutual information of check nodes. With $I_{EV}(i,j)$, we compute the mutual information $I_{EC}(i,j)$ from c_i to its connected variable nodes v_j by using the J function.

4) Decoding threshold evaluation. We finally calculate *a posteriori* mutual information $I_{AP}(j)$, for v_j. We use $J_R(\sigma_{EB}, \sigma_{AP})$ to compute $I_{AP}(j)$ where σ_{AP} is the variance of the summed LLR messages from the check nodes. The evaluation process stops when either $I_{AP}(j) = 1$ for all variable nodes, or the algorithm reaches the maximum number of iterations. The decoding threshold is the lowest value of Λ_b for which $I_{AP}(j) = 1$ for all variable nodes.

B. Asymptotic Weight Distribution Analysis

This section computes the normalized logarithmic AWE $r(\delta)$ to analyze the minimum distance and error floor for the protograph-based LDPC codes. Let $\boldsymbol{\delta}$ denote the scalar normalized total codeword weight, and $r(\boldsymbol{\delta})$ denote the corresponding normalized logarithmic AWE for each vector $\boldsymbol{\delta}$ [19].

Consider a simple $(m, m-1)$ linear block code over GF(q) with a single check node c and m variable node. A $G \times m$ matrix Ω collects all G codewords of the code, with $G = q^{m-1}$. We then obtain a binary $G \times m$ matrix Ω_b by converting all the non-zero elements in Ω into 1. Note that some rows of Ω_b may be the same. Define a $G_r \times m$ binary matrix $\Omega_{b,r}$ that contains all distinct rows of Ω_b. Since the weights of non-zero codeword are between 2 and m, the number of rows in $\Omega_{b,r}$ is $G_r = 2^m - m$, for $q \geq 2$, and $G_r = 2^m - 1$, for $q = 2$. We can approximately calculate $r(\boldsymbol{\delta})$ as

$$r(\boldsymbol{\delta}) = \max_{\{\delta_l : v_l \in V\}} \left\{ \sum_{i=1}^{n_c} a^{c_j}(\omega_j) - \sum_{j=1}^{n_v} (t_i - 1) H_q(\delta_i) \right\}, \tag{6}$$

where t_i is the degree of variable node v_i, and $\sum_{\{\delta_i : v_i \in V\}} \delta_i = \delta$, $H_q(\delta_i) = \delta_i \ln(q-1) + H(\delta_i)$. Here, $H(x)$ is the binary entropy function given by $H(x) = -(1-\delta_i)\ln(1-\delta_i) - \delta_i \ln \delta_i$. In (6), $a^{c_j}(\omega_j)$ is the asymptotic weight-vector enumerator of the constraint node c_j given by

$$a^c(\omega) = \max_{\{P_\omega\}} \left\{ H(P_\omega) + P_\omega \cdot \eta_q^T \right\}, \tag{7}$$

with the constraint that $\{P_\omega\}$ is the set of solutions to $\omega = \{P_\omega\} \cdot \Omega_{b,r}^{\mathcal{C}}$, with $p_1, p_2, ..., p_k \geq 0$ and $\sum_{k=1}^{G_r} p_k = 1$. The vector $\eta_q = [\eta_{q,1}, \eta_{q,1}, ..., \eta_{q,G_r}]$ has entries $\eta_{q,k} = \ln g(q, |\xi_k|)$, where ξ_k is the kth row of $\Omega_{b,r}$, $|\xi_k|$ is the weight of ξ_k, and $g(q,i) = \frac{q-1}{q}[(q-1)^{i-1} + (-1)^i]$.

First, we use the Monte Carlo method to generate random samples for the vector $(\delta_1, \delta_2, ..., \delta_{n_v})$. We then calculate each $a^{c_j}(\omega_j)$ by using a convex optimization method to get the values of $\sum_{i=1}^{n_c} a^{c_j}(\omega_j) - \sum_{j=1}^{n_v}(t_i - 1)H(\delta_i)$, and the largest value among which is $r(\delta)$. Finally, we set $r(\delta)$ for the protograph code ensemble as $n_v r(\delta) = r(\boldsymbol{\delta})$. Note that the ensemble of all rate-R, q-ary (random) linear codes whose parity check matrix entries are i.i.d. uniform has the asymptotic weight enumerator, given by

$$\tilde{r}(\tilde{\delta}) = H_q(\tilde{\delta}) - (1-R)\ln(q), \tag{8}$$

which corresponds to the asymptotic Gilbert-Varshamov (GV) bound for the non-binary case.

IV. The Proposed Protograph Codes

In this section, we propose a fast search approach to obtain good NP-LDPC protographs for 2D-ISI channels. We focus on small Galois fields $GF(q)$ (e.g., $q \leq 16$) since NP-LDPC codes over small fields are more practical in view of the implementation complexity.

Given the same code rate, a larger protograph may have a better decoding threshold and hence a better waterfall region performance due to its larger search space and the design flexibility. However, as protographs grow larger, there is a point of diminishing returns during the optimization of threshold and the search algorithm also becomes more complex. Moreover, for a fixed codeword length, a larger protograph leads to a smaller repetition factor of the protograph, which may cause a higher error floor due to the trapping sets incurred, and vice versa.

Therefore, we first design a small protograph with only one check node. Although the regular non-binary codes with only degree-2 variable nodes designed over large Galois field sizes are known to have good performance [19], we find that through an EXIT and AWE analysis guided search, increasing the variable node degree to more than two for relatively small fields in the protograph can achieve a good balance between the performance of waterfall region and error floor region. As a result, we obtain a small protograph NP1 corresponding to the code rate of 8/9 (see Fig. 3(a)), whose protograph can be represented as

$$\mathbf{B}_{\text{NP1}} = \begin{pmatrix} 2 & 3 & 2 & 3 & 2 & 3 & 2 & 3 & 2 \end{pmatrix}. \qquad (9)$$

Since a larger protograph may yield better decoding thresholds as mentioned earlier, we also design a larger protograph which will enable a lower decoding threshold over 2D-ISI channels. In particular, we first search for a rate-1/2 protograph that contains 3 check nodes and 6 variable nodes. As the corresponding search space is prohibitively large, we propose to start by searching for a protograph with two degree-2 and one degree-3 variable nodes instead, in the form of

$$\mathbf{B}_{\text{NP2}} = \begin{pmatrix} a_1 & a_2 & a_3 & 0 & 1 & 1 \\ a_4 & a_5 & a_6 & 1 & 1 & 1 \\ a_7 & a_8 & a_9 & 1 & 0 & 1 \end{pmatrix}, \qquad (10)$$

where a_i, $i = 1, ..., 9$, are the number of edges connecting the associated variable node and the check node. We further reduce the search space by limiting the number of edges, $a_i \in \{0, 1, 2\}$. In this way, we can achieve a fast search to obtain a resulting protograph

$$\mathbf{B}_{\text{NP2}} = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}. \qquad (11)$$

We then extend the above rate-1/2 protograph to that with higher code rates. With the guide of the EXIT analysis, we obtain the protograph NP2 with a rate $R = (n+1)/(n+2)$ (e.g. $R = 8/9$ with n = 7, see Fig. 3(b)).

Next, we compute the decoding thresholds of the proposed codes by the EXIT analysis described in Section III-A. Fig. 4(a) shows the iterative decoding thresholds of NP1 and NP2 versus different size of $GF(q)$, at rate 8/9 and with the channel matrix corresponding to a 4Tb/in^2 magnetic recording system. Observe that NP1 and NP2 have the lowest thresholds of 3.95 dB and 3.9 dB, respectively, for q =16. In addition,

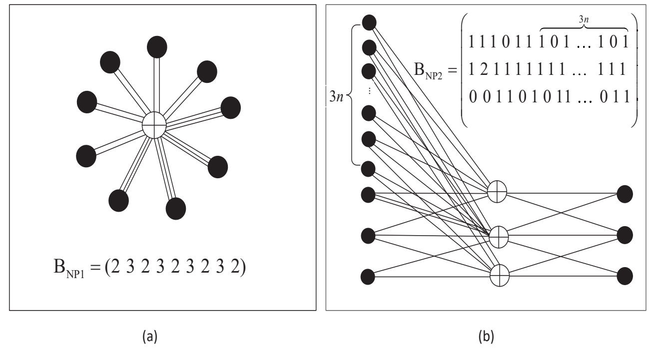

Fig. 3. (a) The proposed protograph NP1, (b) The proposed protograph NP2.

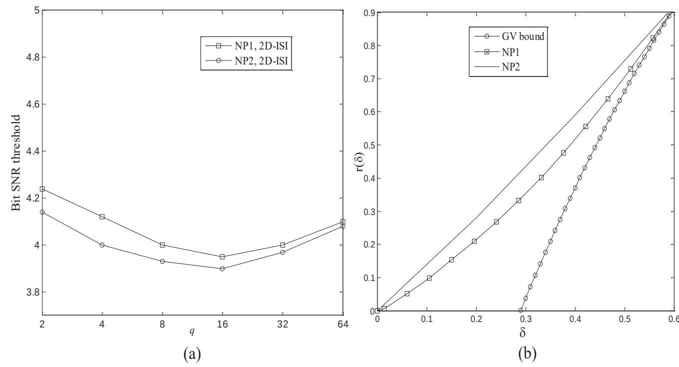

Fig. 4. (a) Bit SNR threshold of the proposed protographs over a 4Tb/in^2 magnetic recording channel, (b) Asymptotic symbol weight enumerators of protographs over GF(16).

we compute the asymptotic AWE for the proposed codes as described in Section III-B. Fig. 4(b) shows the AWE of both protographs and the GV bound for q = 16 and R = 1/2. Observe that the AWE converges to the GV bound as the scalar normalized total codeword weight δ gets larger. The NP1 protograph offers a growth rate closer to the GV bound than NP2, and hence it can provide a better error floor performance.

V. Simulation results

Computer simulations have been carried out to evaluate the performance of the designed NP-LDPC codes over the 4Tb/in^2 magnetic recording channels. Fig. 5 compares the error rate performance of the proposed NP-LDPC codes over $GF(4)$ and $GF(16)$, with a binary irregular LDPC code [13] and a binary protograph-based LDPC code [16], both optimized for the 4Tb/in^2 magnetic recording system. The performance of a NQC LDPC codeword over $GF(16)$ proposed for magnetic recording channels [14] is also included. The code rate is 8/9 and codeword length is 4608 bits for all the codes. The numbers of turbo iteration and LDPC iteration are 10 and 30, respectively. Observe that the NP2 code over $GF(16)$ has the best bit error rate performance at the waterfall region, which coincides with the decoding threshold analysis. The NP1 code over $GF(16)$ outperforms the optimized binary codes by about two orders of magnitude at E_b/N_0 =4.65 dB. It achieves a gain of about 0.15 dB over the binary codes at BER of 10^{-6}. NP1 code also performs better than the NQC code and NP2 code at high SNR region, which is consistent with the AWE analysis. Furthermore, it is observed that the proposed codes are only

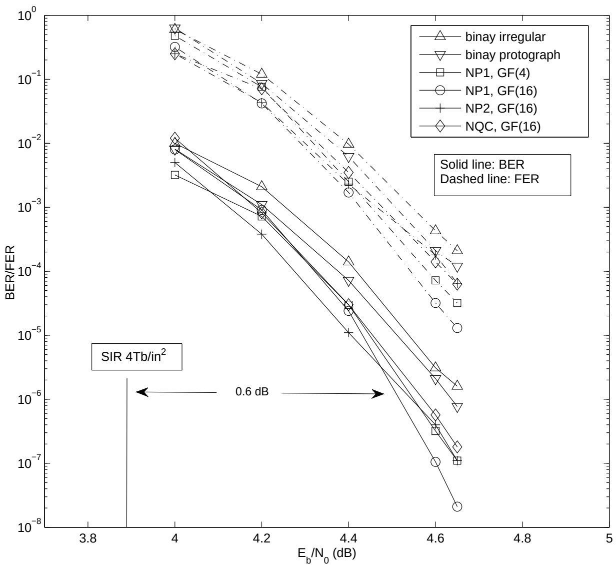

Fig. 5. Performance comparison of the proposed NP-LDPC codes and the existing codes.

about 0.6 dB away from the channel symmetric information rate (SIR) that was defined and computed in [12].

VI. Conclusion

We have considered the construction of NP-LDPC codes for ultra-high density magnetic recording channels with 2D-ISI. In particular, we have first proposed a modified asymptotic EXIT analysis. In conjunction with the AWE analysis, they are used to guide the design of NP-LDPC codes. By further applying a fast search approach, we have constructed two types of NP-LDPC codes for 2D-ISI channels, namely the NP1 code and NP2 code. The EXIT-chart analysis, the AWE analysis, and the error rate simulations have shown that the proposed new codes outperform the previously designed optimized irregular binary LDPC code, the binary protograph code, and the NQC LDPC code, and the corresponding BER performance is only 0.6 dB away from the channel SIR. Moreover, as a benefit of imposing a structural regularity, the NP-LDPC codes can be decoded by means of a semi-parallel architecture, while they render the same encoding complexity increasing linearly with the block length as the NQC codes. Thus, the proposed NP-LDPC codes have shown a great potential for ultra-high density magnetic recording systems.

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