A Detector-aware LDPC Code Optimization for Ultra-High Density Magnetic Recording Channels

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In this paper, a detector-aware low-density parity-check (LDPC) code optimization algorithm is proposed for the ultra-high density magnetic recording channel, such as bit-patterned magnetic recording (BPMR) and two-dimensional magnetic recording (TDMR), where a 2D detector (2D-DET) is employed to combat 2D-ISI effects, by acquiring the variance of the 2D-ISI channel log-likelihood ratios (LLRs) corresponding to the specific 2D-DET. The new parameter builds a bridge of LDPC code optimization between the ISI-channel and AWGN channel. Numerical results show that our proposed algorithms are more efficient in running time than another recently proposed optimization algorithms. Moreover, the protograph based quasi-cyclic (QC) codes using the proposed optimization strategy, which have a low-complexity QC encoder structure with an effectual parallelizable protograph decoder composition, enjoy up to 1 order of magnitude performance gain in bit error rate (BER) over the other random codes that suffer from high error floors.

***Index Terms*—low-density parity-check (LDPC) codes, protograph LDPC codes, quasi-cyclic (QC) LDPC codes, two dimensional (2D) intersymbol interference (ISI) channels**

I. INTRODUCTION

AIMING at further increasing the data-storage density beyond 1Tb/in^2[1], bit-patterned media recording (BPMR), shingled writing (SW)/two-dimensional magnetic recording (TDMR) have been emerging as new promising technologies. However, with the reduction of the track pitch, inter-track interference (ITI) becomes a major impairment for these magnetic recording systems. ITI combined with the conventional (down-track) inter-symbol interference (ISI) forms a two-dimensional (2D) ISI that severely degrades the performance of data storage systems [2]-[3].

A number of approaches have been proposed to combat 2D-ISI effects, including the use of 2D detectors (2D-DETs) [4]-[8] that mitigate the interference in 2D directions and advanced error-correcting coding (ECC) schemes. The symbol-based Bahl-Cocke-Jelinek-Raviv (BCJR) [4] 2D-DET provides optimal detection over such channels, but it has prohibitive complexity for the typical 2D data size [5]. In [8], the well known suboptimal 2D-DET, iterative row-column soft decision feedback algorithm (IRCSDFA) [6], has been further simplified using Gaussian approximation.

By applying the turbo principle to 2D-DET and decoder, ECC also provides a significant means to combat 2D-ISI [9]-[14]. Low-density parity-check (LDPC) code that is well-known for its capacity-approaching ability in the additive white Gaussian noise (AWGN) channel is believed to be indispensable for dense magnetic recording. Existing LDPC codes optimized for symmetric, AWGN-like channels are not optimal for magnetic recording channels due to channel asymmetry. In [9], the authors designed LDPC codes optimized for partial response channels (PRCs) by means of a density evolution (DE) analysis. Recent works [10] provide a comprehensive analysis of the error floor along with code optimization guidelines for structured and regular non-binary (NB) LDPC codes in magnetic recording channels.

Not much work has been done on LDPC codes optimized for 2D-ISI channels. In [11], new LDPC codes optimized for 2D-ISI channels detected using the symbol-based BCJR 2D-DET have been designed, and are shown to achieve better performance than the LDPC codes optimized for AWGN channels. However, the LDPC codes in [11] were designed for use with the full 2D-DET, which is highly complex. LDPC codes optimized for 2D-ISI channels detected using the IRCSDFA-GA detector in [12], exhibit excellent error-correction capabilities, especially for high codeword lengths. However, the ISI-optimized LDPC codes are designed for a 2D-ISI channel detected by the specific 2D-DET, and code optimization procedure should be operated again for a new detector. Moreover, the 2D-DET Extrinsic Information Transfer (EXIT) curve cannot be expressed in the closed form, it is required to statistically average over a large number of 2D detection by Monte Carlo simulation, and the computational burden remains a bottleneck of the optimization approaches.

In this paper, we present a detector-aware LDPC code optimization algorithm for a 2D-ISI channel, acquiring the variance of the channel log-likelihood ratios (LLRs) corresponding to the specific 2D-DET. The new parameter which captures the error characteristics of magnetic recording builds a bridge of LDPC code optimization between the ISI-channel

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and AWGN channel. According to the proposed approach that has lower computational complexity, the existing code optimization techniques previously developed for the AWGN channel which are not effective can be re-used in 2D-ISI channels.

The rest of this paper is organized as follows. Section II presents the 2D-ISI channel model. Section III introduces the detector-aware LDPC code optimization for 2D-ISI channels and then the computational complexity is analyzed. The proposed schemes are validated in Section IV. Finally, conclusions are drawn in Section V .

II. 2D-ISI CHANNEL MODEL

The 2D channel response before detection can be represented by an $L_M \times L_N$ channel response matrix H_{2D}:

$$H_{2D} = \begin{pmatrix} h_{(0,0)} & h_{(0,1)} & \cdots & h_{(0,L_N-1)} \\ h_{(1,0)} & h_{(1,1)} & \cdots & h_{(1,L_N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{(L_M-1,0)} & h_{(L_M-1,1)} & \cdots & h_{(L_M-1,L_N-1)} \end{pmatrix} \quad (1)$$

where L_M and L_N denote the interference lengths in the vertical and horizontal directions, respectively.

The binary data distributed in an array with M rows and N columns is denoted by $x(i,j) \in \{-1,+1\}$, with $i = 1,2,\cdots,M$, and $j = 1,2,\cdots,N$. Then, the received signal at position(i,j) can be expressed by

$$r(i,j) = \sum_{m=0}^{L_M-1}\sum_{n=0}^{L_N-1} h(m,n)x(i^*,j^*) + w(i,j) \quad (2)$$

where $i^* = i - m + d_M$ and $j^* = j - n + d_N$. Here, d_M and d_N are non-causality offsets, and $w(i,j)$ is the AWGN with zero mean and variance σ_w^2. In the coded 2D-ISI channel, the signal-to-noise ratio (SNR) per information bit is defined as

$$E_b/N_0 = 10 \times log_{10}\left(\sum_{m=0}^{L_M-1}\sum_{n=0}^{L_N-1} |h(m,n)|^2 / 2\sigma_w^2 / R_c\right) \quad (3)$$

where R_c is the code rate.

Normally, the offsets are set to be $d_M = \frac{L_M-1}{2}$ and $d_N = \frac{L_N-1}{2}$. It is also assumed that a boundary of -1 around the data block for initializing the 2D-DET, i.e., $x(i,j) = -1$ for $(i,j) \notin \{(i,j)\,|1 \le i \le M, 1 \le j \le N\}$.

III. A DETECTOR-AWARE LDPC CODE OPTIMIZATION

In this section, we present a detector-aware LDPC code optimization approach for 2D-ISI channels and compare the computational complexity with the methods in [11]-[12].

A. *Detector-aware LDPC Code Optimization*

In the 2D-ISI channel, the soft information messages generated by the 2D-DET are exchanged within the LDPC decoder iteratively to perform joint detection and decoding, where the block diagram is given in Fig.1. In the code design procedure [11], the effective mutual information of variable node decoder (VND) combined with the 2D-DET $I_{E,VND}$ is

$$I_{E,VND} = \sum_{i=1}^{D_L} \lambda_i \times I_{E,VND}^{d_v^i} = \sum_{i=1}^{D_L} \lambda_i \times J\left(\sqrt{(d_v^i - 1) \times [J^{-1}(I_{A,VND})]^2 + [J^{-1}(I_{E,DET}^{d_v^i})]^2}\right) \quad (4)$$

where $\lambda_i = \frac{d_v^i}{(1-R_c)\times d_c} \times \varsigma_i$, ς_i is the fraction of nodes incident to variable nodes of degree d_v^i, d_c denotes the check node degree, $J(\cdot)$ and $J^{-1}(\cdot)$ are given in [11], and $I_{E,VND}^{d_v^i}$ denotes the combined 2D-DET and VND Extrinsic Information Transfer (EXIT) function of a degree$-d_v^i$ variable node.

Fig.2 shows the conditional probability density functions (PDFs) of the output LLRs of symbol-based BCJR 2D-DET in an information bit density of 4Tb/in^2 channel using Monte Carlo simulations, where a LDPC code with rate of 8/9 is employed. It is easy to observe that the re-modeled symmetric Gaussian distribution with mean k and variance $\varpi = 2k$ and the LLRs of 2D-DET are well matched for transmitted coded bit 0s and 1s, respectively. Thus, we conclude that the LLRs of a 2D-DET can be approximated by a symmetric Gaussian distribution giving a specific 2D-DET and the EXIT chart can be applied to optimize LDPC codes for such 2D-ISI channels. Then the (4) can be written as (5),

$$I_{E,VND} = \sum_{i=1}^{D_L} \lambda_i \times J\left(\sqrt{(d_v^i - 1) \times [J^{-1}(I_{A,VND})]^2 + \varpi}\right) \quad (5)$$

where ϖ is the estimated variance of output LLRs of 2D-DET.

The LDPC code optimization problem can be accomplished by means of linear programming [11]. Following the proposed procedure, protograph-based QC-LDPC codes can be constructed according to the optimized degree sequences for 2D-ISI channels, where the QC structure is maintained while maximizing the girth of the Tanner graph in the progressive edge-growth (PEG) algorithm [11]. The proposed strategy of detector-aware codes design is summarized in Algorithm 1.

B. *Computational complexity analysis*

The symbol-based BCJR 2D-DET trellis has $2^{M(L_N-1)}$ states and 2^M input and output branches for each state, therefore its complexity is $O(N \cdot 2^{ML_N})$. The complexity of the IRCSDFA-GA, whose state space dimension of the ISI trellis of either component detector is reduced enormously by employing the GA, is $O(I_{CD} \cdot MN \cdot ((L_M-1)L_N 2^{L_N} + L_M(L_N-1)2^{L_M})$, where I_{CD} denotes the iteration number between two component detectors in the IRCSDFA-GA detector [8]. In [11]-[12], the 2D-DET EXIT curve cannot be expressed in the closed form, so it is essential to statistically average over a huge number of 2D detection simulations. Specifically, once the a-prior mutual information $I_{A,DET}$ and 2D-ISI channel output sequence $\mathbf{r}$ are generated, the 2D-DET is run. The 2D-DET output LLRs are used to calculate the $I_{E,DET}$ (see [11]), which is measured by estimating the $p(\zeta_{|x=1})$ and $p(\zeta_{|x=-1})$ from a histogram of the extrinsic information values, where ζ denotes the element of LLRs.

Algorithm 1 Detector-aware LDPC code design algorithm

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Initialize: code rate $R_c$, code length $N_c$, initial channel $E_b/N_0$, the step $\Delta_s$, target cost $T_{cost}$. $\lambda = (\lambda_2, \lambda_3, \cdots, \lambda_{D_L})$ and $\rho = (\rho_2, \rho_3, \cdots, \rho_{D_R})$.
Run the 2D-DET once and calculate the a-posteriori mutual information of the 2D-DET.
Model the distribution of LLRs as an Gaussian with variance $\varpi$
Based on (5), select the best $\lambda$ sequences to fit the check node decoder (CND) EXIT curve as closely as possible to the EXIT curve of the combined 2D-DET and VND.
Design LDPC parity check matrix according the optimized degree sequences. Go to next step if protograph-based QC-LDPC code is to be presented, or END.
Adjust the degree sequence based on the code length $N_c$, code rate $R_c$ and replication fact $J$, and obtain the protograph base matrix $\mathbf{H_p}$ ($L_{pm} \times L_{pn}$).
Permute the edges of the nodes in the $J$ replicas of the base protograph matrix $\mathbf{H_p}$:
for $i = 1$ to $L_{pn}$ do
  for $j = 1$ to $J$ do
    if (j == 1) then
      Select the check nodes with larger local girth based on the modified PEG algorithm
    else
      //Cyclically shift others to get the new position $L_j^l$.
      for $l = 1$ to $N_i^d$ do
        //$N_i^d$ is the number of connections of the check nodes connected with nodes $v^i$ of $\mathbf{H_p}$.
        $L_j^l = \frac{L_{j-1}^l}{J} \times J + \left(L_{j-1}^l + 1\right) \mod J$
      end for
    end if
  end for
end for
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TABLE I
COMPUTATIONAL COMPLEXITY OF DIFFERENT OPTIMIZATION SCHEMES

Optimization schemes	Computational complexity
2D BCJR-based Opt. [13]	$O(L_\lambda \cdot L_\hbar \cdot N \cdot 2^{ML_N})$
Proposed BCJR-aware Opt.	$O(N \cdot 2^{ML_N})$
2D GA-based Opt. [14]	$O(L_\lambda \cdot L_\hbar \cdot I \cdot MN \cdot ((L_M - 1)L_N 2^{L_N}$ $+L_M(L_N - 1)2^{L_M}))$
Proposed GA-aware Opt.	$O(I \cdot MN \cdot ((L_M - 1)L_N 2^{L_N}$ $+L_M(L_N - 1)2^{L_M}))$

Instead of running the 2D-DET $L_\lambda \cdot L_\hbar$ times, we apply the 2D-DET once in the proposed optimization scheme, where L_λ is the interval of $I_{A,DET}$ and $L_\hbar$ is the 2D-DET running time for the histogram to estimate the $p(\zeta_{|x=1})$ and $p(\zeta_{|x=-1})$. The computational complexities of the 2D BCJR-based optimization, 2D GA-based optimization, and corresponding detector-aware ones are listed in Table I which are for off-line optimization of the designed codes.

TABLE II
THE OPTIMIZED DEGREE SEQUENCES OF LDPC CODES

	λ_2	λ_3	λ_7	λ_8	λ_9	λ_{10}	λ_{15}
The proposed	0.0644	0.5953	0.2267	0.1136			
Scheme 1	0.1445	0.8146			0.0378	0.0031	
Scheme 2	0.0622	0.4016					0.5362

IV. SIMULATION RESULTS

In this section, we present the simulation results with the LDPC schemes proposed in Section III. The symbol-based BCJR-aware code optimization is proposed and verified for a 4Tb/in^2 2D-ISI channel, however, it can easily be extended for other 2D detector-aware schemes. In the simulations, we set the maximum number of iterations between the symbol-based BCJR 2D-DET and the LDPC belief-propagation (BP) decoder to 10, and the maximum number of BP iterations in LDPC decoder to 30, all codes have rate of $R_c = 8/9$. The 2D channel response matrix (CRM) is assumed to be (6), corresponding to a magnetic recording density of 4Tb/in^2 [11].

$$H_{4T} = \begin{pmatrix} 0.0368 & 0.1435 & 0.0368 \\ 0.2299 & 0.8966 & 0.2299 \\ 0.0368 & 0.1435 & 0.0368 \end{pmatrix} \tag{6}$$

In Fig.3, we compare the BER performance of our proposed LDPC codes with the 2D-ISI-optimized (Scheme 1) [11] and AWGN-optimized (Scheme 2) for two different code lengths in a 2D-ISI channel with recording density of 4Tb/in^2, where the optimized degree sequences of the LDPC codes are given in Table II. Fig.3 shows that the proposed LDPC codes (red lines) has a performance gain of about 0.2 dB over the codes of Scheme 2 (blue dashed lines) at BER of 10^{-5} for all two different code lengths ($Nc = 4608, 9216$) in a 2D-ISI channel with recording density of 4Tb/in^2. Furthermore, there is only small loss in performance of the proposed codes when compared to the codes which are designed by the significantly more complex optimization approach (Scheme 1) at BER of 10^{-5} for all two different code lengths, and the proposed codes have a lower error floor at high SNR region.

To further evaluate the validity of the proposed detector-aware LDPC code optimization algorithm, we design the protograph based QC-LDPC code according to Algorithm 1 as described in Section III. Fig.4 shows the BER comparison between the protograph-based QC-LDPC code and random-like protograph LDPC codes [15]. As shown, the proposed protograph based QC-LDPC code is able to provide up to 1 order of magnitude performance gain relative to the other random codes that suffer from high error floor, and no error floor is observed down to a BER of about 10^{-8}. Note that the protograph based QC structure only causes a slight performance degradation compared with others.

V. CONCLUSION

A low-complexity detector-aware LDPC code optimization algorithm has been proposed for dense 2D-ISI magnetic recording channels. The new variance of the channel LLRs

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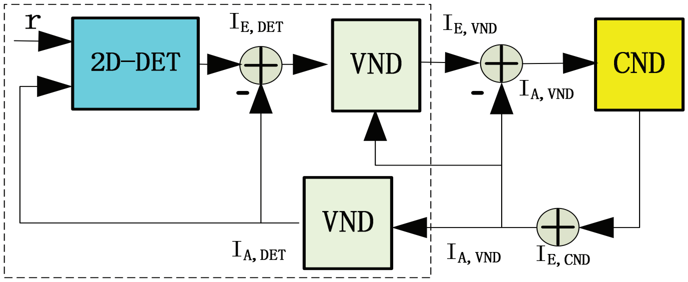

Fig. 1. Block diagram of joint 2D-DET and decoding for a 2D-ISI channel.

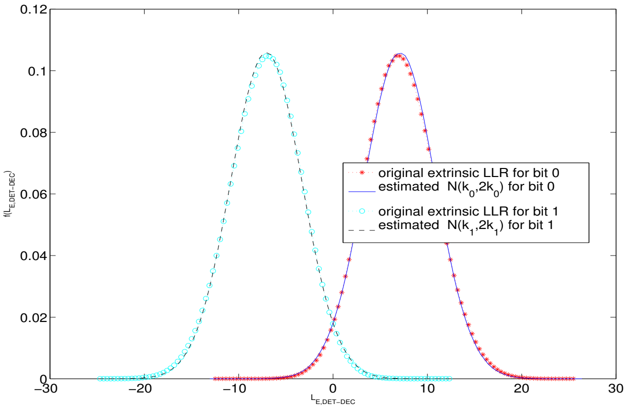

Fig. 2. PDFs of the 2D-ISI channel L-values detected by symbol-based BCJR 2D-DET in a 4Tb/in^2 magnetic recording channel with $E_b/N_0 = 4dB$.

corresponding the specific 2D-DET captures the error characteristics of 2D-ISI magnetic recording channels. Based on the detector-aware code optimization approach, new LDPC codes have been designed for 4Tb/in^2 channels. Simulation results have verified that our proposed protograph based QC-LDPC codes can perform as well as the random LDPC codes, obtain up to 1 order of magnitude performance gain over other random codes, and no error floor is observed down to a BER of about 10^{-8}, while retaining the implementational benefits.

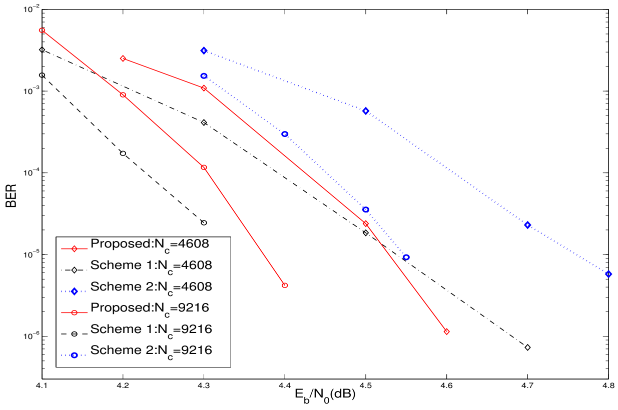

Fig. 3. BER performance comparison with different schemes in a 4Tb/in^2 channel employed the same symbol-based BCJR 2D-DET.

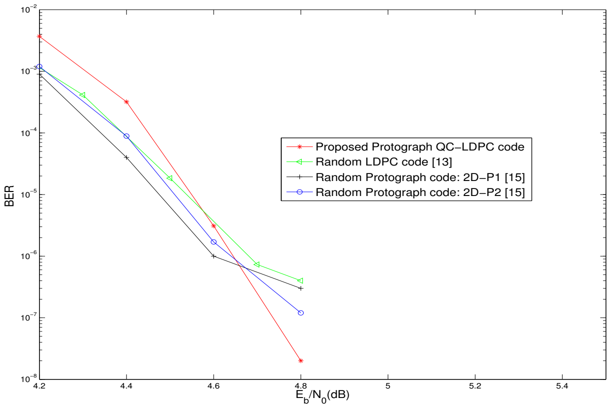

Fig. 4. BER performance for the protograph-based QC-LDPC code and random-like LDPC codes in a 4Tb/in^2 channel.

REFERENCES

[1] R. Wood, M. Williams, A. Kavcic, and J. Miles, "The feasibility of magnetic recording at 10 Tb/in2 on conventional media," *IEEE Trans. Magn.,* vol. 45, no. 2, pp. 917-923, Feb. 2009.

[2] Y. Wang, B. V. K. Vijaya Kumar, "Multi-track joint detection for shingled magnetic recording on bit patterned media with 2-D sectors," *IEEE Trans. Magn.,* vol. 52, no. 7, Article#: 3001507, Jul. 2016.

[3] J. Yao, E. Hwang, B. V. K. Vijaya Kumar, et al., "Two-track joint detection for two-dimensional magnetic recording (TDMR),"in *Proc. IEEE ICC,* London, UK, Jun. 8-10, 2015, pp.418-424.

[4] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory,* vol. 20, no. 2, pp. 284-287, Mar. 1974.

[5] B. M. Kurkoski, "Towards efficient detection of two-dimensional intersymbol interference channels," *IEICE Trans. Fundamentals,* vol. E91-A, no. 10, Oct. 2008.

[6] T. Cheng, B. J. Belzer, and K. Sivakumar, "Row-column soft-decision feedback algorithm for two-dimensional intersymbol interference," *IEEE Signal Processing Lett*., vol. 14, no. 7, pp. 433-436, Jul. 2007.

[7] J. Yao, K.C. Teh, and K. H. Li, "Joint iterative detection/decoding scheme for discrete two-dimensional interference channels," *IEEE Trans. Commun*., vol. 60, no. 12, pp. 675-681,Dec. 2012.

[8] J. Zheng, X. Ma, Y. L. Guan, K. Cai, and K. S. Chan, "Low-complexity iterative row-column soft decision feedback algorithm for 2D inter-symbol interference channel detection with Gaussian approximation,"*IEEE Trans. Magn.,* vol. 49, no. 8, pp. 4768-4773, Jun. 2013.

[9] A. Kavcic, X. Ma, and M. Mitzenmacher, "Binary intersymbol interference channels: Gallager codes, density evolution and code performance bounds," *IEEE Trans. Inform. Theory,* vol. 49, no. 7, pp. 1636–1652, Jul. 2003.

[10] A. Hareedy, B. Amiri, R. Galbraith, and L. Dolecek, "Non-Binary LDPC Codes for Magnetic Recording Channels: Error Floor Analysis and Optimized Code Design," *IEEE Trans. Commun*., vol. 64, no. 8, pp. 3194-3207, Aug. 2016.

[11] L. Kong, Y. L. Guan, J. Zheng, G. Han, K. Cai, and K. S. Chan, "EXIT-chart-based LDPC code design for 2D ISI channels," *IEEE Trans. Magn.,* vol. 49, no. 6, pp. 2823-2826, Jun. 2013.

[12] Y. L. Guan, G. J. Han, L. J. Kong, K. S. Chan, K. Cai and J. Zheng, "Coding and signal processing for ultra-high density magnetic recording channels," in *Proc. Int. Conf. on Comp., Networking and Commun. (ICNC),* Hawaii, USA, Feb. 3-6, 2014, pp. 194-199.

[13] E. Hwang, R. Negi, and B. Kumar, "Additive encoding low-density parity-check (AE-LDPC) codes for two-dimensional magnetic recording (TDMR)," in *Proc. Int. Conf. on Comp., Networking and Commun. (ICNC),* Hawaii, USA, Jan. 30-Feb. 2, 2012, pp. 481-485.

[14] M. Mehrnoush, B. J. Belzer, K. Sivakumar and R. Wood, "EXIT chart-based IRA code design for TDMR turbo-equalization system," *IEEE Trans. Commun.,* vol. 65, no. 4, Apr. 2017.

[15] P. P. Chen, L. J. Kong, Y. Fang, and L. Wang, "The design of protograph LDPC codes for 2-D magnetic recording channels," *IEEE Trans. Magn.,* vol. 51, no. 11, Article#: 3101704, Nov. 2015.